

Re: Fibonacci[1,000,000,000] contains 208,987,640 decimal digits

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In article <2npfirF385mqU1@uni-berlin.de>,

Michael Taktikos <michael.taktikos@hanse.net> wrote:

> "Ed Pegg Jr" <edpegg@gmail.com> wrote in *comp.soft-sys.math.mathematica*:

>> *For finding the number of digits in any sufficiently large Fibonacci*

>> *number in base 10, let $k = (\text{ArcCsch}[2])/\text{Log}[10]$. The number of*

>> *digits in Fibonacci[n] is Round[n k].*

>>

>> $k \sim 0.20898764024997873376927208923755541682245923991821$

As someone else already pointed out, this is out by one a significant number of times. The version I would use (for $n > 0$) is:

$$\text{Ceiling}(n \cdot \text{Log}[10](\phi) - \text{Log}[10](\text{Sqrt}(5)))$$

where $\phi = (1 + \text{Sqrt}(5))/2$. $\text{Log}[10](\phi)$ is the value of k above, and $\text{Log}[10](\text{Sqrt}(5))$ is approximately 0.349485+; thus the discrepancy.

>> *Fibonacci[10^9] has $10^9 k \sim 208987640$ digits.*

>

> *Thank you for confirming this result. It means, that the number mentioned in Sloane's sequence A068070 (number of digits of the 10^n -th Fibonacci number), like some other numbers in this sequence, are wrong.*

> *N. Sloane get now a comment to A068070 with the corrected sequence*

> *{0,2,21,209,2090,20899,208988,2089877,20898764,208987640,2089876402,*

> *20898764025,208987640250,2089876402500,20898764024998,208987640249979,*

> *2089876402499787,20898764024997873,208987640249978734,...}*.

The first value is clearly not correct — the formula does not work for $n = 0$ as the contribution from $1/\phi^n$ in the usual formula is not negligible. Your values for $n = 10$ and $n = 17$ are also incorrect; they should be 2089876403 and 20898764024997874 respectively. The others are correct.

sci.math.symbolic: Re: Fibonacci[1,000,000,000] contains 208,987,640 decimal digits

[The current data in the online encyclopaedia as I write this is wrong for $n = 8, 9$ and 16 .]

Cheers,
Geoff.

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