

Animation of rotating axes

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I am reading an introductory textbook on theoretical mechanics. In the chapter on moving coordinate systems the author sets a problem to solve:

An xyz coordinate system is rotating with respect to an XYZ coordinate system having the same origin and assumed to be fixed in space (i.e. it is an inertial system).

The angular velocity of the xyz system relative to the XYZ system is given by $w = 2t\mathbf{i} - t^2\mathbf{j} + (2t+4)\mathbf{k}$ where t is the time.

The position vector of a particle at time t as observed in the xyz system is given by $r = (t^2+1)\mathbf{i} - 6t\mathbf{j} + 4t^3\mathbf{k}$.

Find

- a) the apparent velocity and
- b) the true velocity at time $t=1$.

Answer: (not relevant to this posting, included for completeness)

a) Apparent velocity at time t is $dr/dt = 2t\mathbf{i} - 6\mathbf{j} + 12t^2\mathbf{k}$.
At time $t=1$, this is $2\mathbf{i} - 6\mathbf{j} + 12\mathbf{k}$

b) True velocity at time t is $dr/dt + w \times r = 34\mathbf{i} - 2\mathbf{j} + 2\mathbf{k}$ at $t=1$

I am trying to use animation in MuPad to demonstrate visually the full motion of the particle, by showing its motion relative to the xyz coordinate system and the motion of the xyz coordinate system relative to the XYZ system.

Using MuPad, the motion of the particle relative to xyz is simply...

```
x_axis := plot::Arrow3d([1, 0, 0], Color = RGB::Red):
y_axis := plot::Arrow3d([0, 1, 0], Color = RGB::Red):
z_axis := plot::Arrow3d([0, 0, 1], Color = RGB::Red):
r := plot::Arrow3d([t^2+1, 6*t, 4*t^3], t = 0..1, Color = RGB::Blue):
plot(x_axis, y_axis, z_axis, r);
```

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Next, I need to use `plot::Rotate3D` to rotate this whole system with angular velocity $w = 2t\mathbf{i} - t^2\mathbf{j} + (2t+4)\mathbf{k}$

`plot::Rotate3d` is described as:

`plot::Rotate3d(angle, [cx, cy, cz], [dx, dy, dz], object)` rotates a 3D object by the given angle around the rotation axis defined by the point $[cx, cy, cz]$ and the direction $[dx, dy, dz]$.

The angular velocity w in the problem is specified relative to the xyz system. It appears to me that, to use `Rotate3d` to rotate the system, I need W which is w relative to the XYZ system.

In the XYZ system, let the unit vectors along the axes be I, J, K and $W = W1I + W2J + W3K$. Since w and W are the same vector $w = W$, so that

$$W1I + W2J + W3K = 2t\mathbf{i} - t^2\mathbf{j} + (2t+4)\mathbf{k}$$

Successively take the dot product of both sides with I, J and K to obtain

$$W1 = 2t\mathbf{i} \cdot \mathbf{i} - t^2\mathbf{j} \cdot \mathbf{i} + (2t+4)\mathbf{k} \cdot \mathbf{i}$$

$$W2 = 2t\mathbf{i} \cdot \mathbf{j} - t^2\mathbf{j} \cdot \mathbf{j} + (2t+4)\mathbf{k} \cdot \mathbf{j}$$

$$W3 = 2t\mathbf{i} \cdot \mathbf{k} - t^2\mathbf{j} \cdot \mathbf{k} + (2t+4)\mathbf{k} \cdot \mathbf{k}$$

$\mathbf{i} \cdot \mathbf{i}$ is the cosine of the angle between \mathbf{i} and \mathbf{i} and similarly for the other dot products.

This gives me the components of W , i.e. $W1, W2$ and $W3$, but not in a form I can use with `Rotate3D`.

Every idea I have tried has not born any fruit.

If anyone has some advice on how to perform this animation it would be much appreciated.

Cheers,
Brad

• ***Follow-Ups:***

◆ ***Re: Animation of rotating axes***

◇ *From:* Christopher Creutzig

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