Re: divisors of zero

Source: http://sci.tech−archive.net/Archive/sci.math.symbolic/2006−05/msg00013.html

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- *From*: Clifford Nelson <cjnelson9@xxxxxxxxxxx>
- *Date*: Tue, 02 May 2006 18:26:35 GMT

In article <e37vb3$5kd$1@xxxxxxxxxxxxxxxxx>,
rusin@xxxxxxxxxxxxxxxxxxxxxx (Dave Rusin) wrote:

> In article <e33plqllf1@xxxxxxxxxxxxxxxxxxxxxxx>,
> Robert Israel <israel@xxxxxxxxxxx> wrote:
>
>> In article <cjnelson9−852E3E.15440530042006@xxxxxxxxxxxxxxxx>,
>> Clifford Nelson <cjnelson9@xxxxxxxxxxx> wrote:
>>
>>> http://groups.google.com/group/geometry.research/browse_frm/thread/23704a
>>> ecd086c689/e365d3da1264633d?lnk=st&q=4D+field&rnum=12#e365d3da1264633d
>>>
>>> If you can get the URL above to work you can see that Robin
>>> Chapman and
>>> John Rickard found divisors of zero for B_5 numbers in
>>> about a day. I
>>> don't know how they found them. How would you find them
>>> with
>>> Mathematica?
>>
>> I don't know what they did, either. But here's what I might do in Maple.
>> This is the formula for (a,b,c,d)*(e,f,g,h), quoted in Chapman's article:
>>
>>> L:= [1/5*((a + b + c + d)*e − d*f − c*g − a*(−e − f − g − h)
>>> −
>>
>> b*h), 1/5*(−a*e + (a + b + c + d)*f − d*g −
>> b*(−e − f − g − h) − c*h),
>> 1/5*(−b*e − a*f + (a + b + c + d)*g −
>> c*(−e − f − g − h) − d*h),
>> 1/5*(−c*e − b*f − a*g − d*(−e − f − g − h) +
>> (a + b + c + d)*h)];
>
> This product formula shows that when we allow this algebra to act on
> itself by left−multiplication, we get a linear embedding of the

algebra into the 4x4 matrix ring. (That is, it's an embedding of vector spaces; such a product formula defines an embedding of rings iff the associative law holds.) Then x = (a,b,c,d) is a zero−divisor iff the corresponding linear map has determinant zero. Well, the determinant is a homogeneous polynomial of degree 4 in a,b,c,d (it's actually monic in each variable, after multiplying by 125). So setting it equal to zero defines a projective surface; you just want to know whether there are any points on that surface.

If the algebra is defined over an algebraically closed field like C, then the answer is clearly yes (pick any a,b,c and solve for d). If the base field is the reals this is not so obvious but some random experimentation finds plenty of points (e.g. it's easy to make the discriminant w.r.t. d vanish, and then find points). Most of the simple things I tried let to the golden ratio or 5th roots of unity in some way.

I looked for a little while to find a rational point on this surface but was not successful. It may be that there is a simple modular restriction disallowing this; with the previous paragraph in mind I guess I might try to show there are no 5−adic points if I thought there were no rational ones. (E.g. this works to show there are no points with a=d.) But I didn't spend much time on this.

dave

Is my memory playing tricks on me or do I remember getting and verifying answers in Mathematica of LinearSolve[M,v] for n by n matrices M and length n vectors v even when M did not have an inverse and had a determinant of zero?

Cliff Nelson

Dry your tears, there's more fun for your ears, "Forward Into The Past" 2 PM to 5 PM, Sundays, California time, at: http://www.kspc.org/
Don't be a square or a blockhead; see:
http://bfi.org/node/574
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