Re: euclidean algorithm over Q[i]

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- *From*: "Chip Eastham" <hardmath@xxxxxxxxx>
- *Date*: 15 Dec 2006 20:57:23 −0800

G. A. Edgar wrote:

> In article <62tgh.6569$Dr3.1078@xxxxxxxxxxxxxxxxxxxx>, Jeremy Watts
> <stevie4545@xxxxxxxxxxx> wrote:
>
>> how does the euclidean algorithm proceed for numbers in Q[i] ? my
>> knowledge
>> of abstract algebra's basic to say the least so i hope i am using the
>> correct term. i mean for complex numbers with rational real & complex parts
>> ie. of form a/b + c/d i a,b,c,d in Z
>>
>> i'm sure i have seen this somewhere but cant find reference to it anywhere.
>>
>> thanks
>
> Q[i] is a field, so greatest common divisors are easy.
> Any nonzero element divides any other nonzero element.
>
> Z[i], the Gaussian integers, may be what you remember seeing.
>
> −−
> G. A. Edgar http://www.math.ohio−state.edu/~edgar/

And if the OP is interested in the Gaussian integers Z[i],
it is helpful to bear in mind that a Euclidean domain is
possessed of a norm. The "remainder" term is to have
norm less than the divisor in each application of the
division algorithm. Of course Z[i] is not an ordered ring,
so its important to pick the remainder to have minimum
norm as its defining characteristic.

The norm of z = a + bi is a^2 + b^2 in Z[i].

More discussion here:

http://en.wikipedia.org/wiki/Gaussian_integer

regards, chip

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