solutions, missing and excess, to transcendental equations [Re: JB discovers yet another K−12 level Maple bug the long liver, 1992−−2007−−? (solve)]

Source: http://sci.tech−archive.net/Archive/sci.math.symbolic/2007−06/msg00051.html

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- *Date*: Fri, 15 Jun 2007 10:50:09 −0700

On Jun 15, 6:07 am, Vladimir Bondarenko <v...@xxxxxxxxxxxxxxx> wrote:

> ....
>
> JB describes here his QA−valuable effort.
>
> http://groups.google.com/group/comp.soft−sys.math.maple/msg/36c0a1d1d...

Looks like about 85% rant (perhaps I underestimate). I see no value in that. Not even as entertainment.

> ....
> BUG # XXXXX solve: INVALID ROOT VALUE
>
> REGRESSION: NO
>
> REPRODUCIBLE: ALWAYS
>
> BUG HISTORY:
>
> PRESENT Maple 11.00,IBM INTEL NT, Feb 16 2007 Build ID 277223
> [...]
> DESCRIPTION: During 15+ years Maple returns a set of two roots,
> one of them false, for a trivial K−12 equation.
>
> TEST CASE: solve(sin(arctan(z)+arctan(2*z)) = 1/sqrt(2), z);
>
> ACTUAL: z = −3/4+1/4*17^(1/2), z = −3/4−1/4*17^(1/2)
>
> z = .2807764060, z = −1.780776406
>
> EXPECTED: z = −3/4+1/4*17^(1/2)

CHECKUP: evalf(subs(z = −3/4−17^(1/2)/4, sin(arctan(z)
+ arctan(2*z)) = 1/sqrt(2)));

−.7071067806 = .7071067810
[...]
Best wishes,

Vladimir Bondarenko
[...]

I don't see this example is in any sense trivial. I will discuss what I think are the salient points.

Math program "solve" functions tend to rely on reducing input to a set of "polynomials" in some set of "variables". Where transcendental functions are involved this will require encapsulating them, and inverses thereof, in order to unwind the system e.g. via Groebner bases. One runs up against at least two issues.

(1) Where multi−valued "functions" e.g. log are concerned, some solutions may be missed. Depends a lot on how much the program "knows" about finding roots other than that given by, say, principal branch values.

(2) It is quite easy to get spurious, or parasite, solutions. Depending on problem specifics, it may be difficult or impossible to recognize them. Indeed, if parameters are present then which "solution" is valid and which a parasite, and in fact even how many are valid, can depend on parameter values.

To some extent these issues are further influenced by the specifics of the polynomialization process. For example, if trigs are converted to complex exponentials one may get different solutions and different parasites than would be obtained from not doing said conversion.

In Mathematica one can see the "full" set of solutions for the example in question via

In[8]:= InputForm[Solve[Sin[ArcTan[z]+ArcTan[2*z]] == 1/Sqrt[2], z,
VerifySolutions−>False]]

Out[8]//InputForm=
{{z −> (−3 − Sqrt[17])/4}, {z −> (3 − Sqrt[17])/4}, {z −> (−3 +
Sqrt[17])/4},
{z −> (3 + Sqrt[17])/4}}

So we get two valid solutions and two parasites. Weeding out the latter is done by numeric means (since no parameters are present), and to be honest this involves a bit of voodoo. One thing I will claim is

that Mathematica's sometimes−criticized bignum numerics model,
significance arithmetic, tends to work quite well for determining a
value is zero to some given accuracy.

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