

Re: $\exp(x) + kx = 0$; analytic solution

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- *From:* Robert Israel <israel@xx>
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Jean-Marc Gulliet <jeanmarc.gulliet@xxxxxxxx> writes:

Per Abrahamsen wrote:

I can easily convince myself that there is exactly one solution when $k > 0$, but does it have a nice analytical solution, or do I have to use

numeric methods?

I apologize if this is the wrong place to ask.

The analytic solution returned by Mathematica 6.0.1 for k positive and x real is in terms of **ProductLog** (principal value of the Lambert W-function in Mathematica. See <http://mathworld.wolfram.com/LambertW-Function.html>)

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In[1]:= Reduce[{Exp[x] + k x == 0, k > 0}, x, Reals]
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Out[1]= k > 0 && x == -ProductLog[1/k]
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Or in Maple, $x = -\text{LambertW}(1/k)$.

The LambertW function has infinitely many branches, but only the principal branch gives a real solution when $k > 0$. When $k < -e$, there are two real solutions given by the principal branch and the "-1" branch.

When $k = -e$, there is one real solution $x=1$ (common to both those branches).

When $-e < k \leq 0$ there are no real solutions.

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Robert Israel israel@xx
Department of Mathematics <http://www.math.ubc.ca/~israel>
University of British Columbia Vancouver, BC, Canada