Re: Rational approximation of exponential function

Source: http://sci.tech−archive.net/Archive/sci.math/2004−06/3038.html

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From: Rob Johnson (*rob_at_trash.whim.org*)
Date: 06/15/04

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Date: Tue, 15 Jun 2004 02:32:06 +0000 (UTC)
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In article <cal4kbgel1@newslocal.mitre.org>,
lewis@PROBE.mitre.org (Keith A. Lewis) wrote:
>"Stephen J. Herschkorn" <herschko@rutcor.rutgers.edu> writes in article
<40CE0381.1070704@rutcor.rutgers.edu> dated Mon, 14 Jun 2004 19:56:53 GMT:
>>>>In article <caetaq$7d2$1@newslocal.mitre.org>,
>>>>lewis@PROBE.mitre.org (Keith A. Lewis) wrote:
>
>>>>>kariwala@ualberta.ca (Vinay Kariwala) writes in article
>>>>><200406112211.i5BMBDa10382@proapp.mathforum.org> dated Fri, 11 Jun 2004
>>>>>23:33:31 +0000 (UTC):
>>>>>>This problem would be trivial for most of you. I am looking for an
>>>>>>approximation of the exponential function, e^(−x) with the following
>>>>>>properties:
>>>>>>
>>>>>>1. The nth order approximation f(x,n) should be rational, i.e. it
>>>>>>should be possible to write f(x,n) as ratio of polynomials in x. f
>>>>>>(x,n) = g(x,n)/h(x,n), where g(x,n) has less number of zeros than h(x,n).
>>>>>>
>>>>>>2. As n approaches infinity, f(x,n) should converge back to the
>>>>>>exponential function.
>
>>>>>Use the Taylor series. The derivative of e^(−x) is −e^(−x), and e^0=1.
>
>>A assume you mean to let g be the nth order Taylor approximation of x
>>|−> exp(−x) and h be indentically 1. Then g has more zeroes than h.
>
>Yes, that's what I meant. And g does indeed have more zeroes in x, although
>I'm having a hard time seeing why you'd even want zeroes in the denominator
>of an approximation for an exponentially decreasing function.

There are times when a rational approximation is nice. For example,
last month, in response to a post by David Cantrell, I used the rational
approximation e^x = (2+x)/(2−x) while estimating n! to get a refinement
of Stirling's formula

$$n! \sim \sqrt{2 \pi n} \left(\frac{n}{e} \right)^n e^{1/(12n)}$$

$$\sim \mathrm{sqrt}(2\,\pi\, n) \left(\frac{n}{e}\right)^{n} \frac{24n+1}{24n-1}$$

and a refinement of Burnside's formula

$$n! \sim \mathrm{sqrt}(2pi) \left(\frac{n+1/2}{e}\right)^{n+1/2} e^{-1/(24n+12)}$$

$$n! \sim \mathrm{sqrt}(2pi) \left(\frac{n+1/2}{e}\right)^{n+1/2} \frac{48n+23}{48n+25}$$

If you are interested, see

http://groups.google.com/groups?threadm=20040504.032925@whim.org

Rob Johnson <rob@trash.whim.org>
take out the trash before replying