Re: Rational approximation of exponential function

Source: http://sci.tech−archive.net/Archive/sci.math/2004−06/3040.html

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Date: 06/15/04

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Date: 14 Jun 2004 19:41:56 −0700
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kariwala@ualberta.ca (Vinay Kariwala) wrote in message
news:<200406120654.i5C6sBx24359@proapp.mathforum.org>...
> On 11 Jun 2004, Dmitriy Samsonov wrote:
> >> 1. The nth order approximation f(x,n) should be rational, i.e. it
> >> should be possible to write f(x,n) as ratio of polynomials in x. f
> >> (x,n) = g(x,n)/h(x,n), where g(x,n) has less number of zeros than h(x,n).
> >Why do you need this? Just curious... Cause, it's the only reason not to use
> >Taylor series, or am totaly wrong?
> >
> >> Any comments or suggestions will be very helpful.
> >Not in case of me answering, but your comment on my answer can be very
> >helpful:)
> >
> >
>
> My research interest is in the area of control. The optimal control
> theory does not handles time delay systems very well. The time delay
> is often represented as an exponential function for continuous time
> systems. So to derive some particular result, I thought that may be
> I can approximate the time delay by a rational function. Then, solve
> the problem with approximation order n. When n goes to infinity, the
> error in the expressions due to approximation would go to zero, provided the rational approximation converges. This was the motive.
>
> As Oscar pointed out, Pade approximants would do the job. I was thinking along the same lines, but could not find a proof of their convergence. Any suggestions?
>
> Vinay
By the theorem of Montessus de Ballore, the [K/N] Pade approximant of
a meromorphic function for fixed N converges as K goes to inf. Notice
that here the numerator degree increases while that of the
denominator's is fixed.

The approximation of exp(−t) over [0, inf) or any finite segment of
that is a vast subject. The "best" results for rational approximation
of the Chebyshev equiripple type (the uniform error bound) is known to
converge as r^N (r=1/9.28902549.... !!!) where N is the degree of the
denominator polynomial.

See

Trefethen and Gutknecht: " The Caratheodory−Fejer method for rational approximation," SIAM J. Num. Anal., 20 (1983) pp420−436

(an astonishingly beautiful work)

Numerical evidence shows that the so−called Chebyshev−Pade (see: Clenshaw and Lord,"Rational approximations from Chebyshev series," Studies In Numerical Analysis, AP, pp95−111) is much faster converging than the simple Pade sequence.