Re: Comments needed for attempted proof of a simple integral

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From: Chairman of the Ozzy Osbourne Appreciation Society (*mathgeek42_at_hotmail.com*)
Date: 06/16/04

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[adding sci.math for wider audience]

No more comments?

I'm self studying through Tom Apostol's Calculus and it's hard
to know if I'm on the right track or if I've gone off track
without feedback. Usenet is currently my only means of
getting feedback.

Apparently there are no answer books for this text. The standard
"solve X" problems aren't so difficult because standard math
education has been about solving problems of that nature and
I can usually verify answers to those problems with software.
It's proofs that are hardest type of exercise for me to verify,
and there are a lot of those in this book. I've considered switching
to Spivak just for access to answers to those kinds of problems
although I feel that I've already made a large enough investment
into Apostol's book to finish it out, even if that means sacrificing
some degree of certainty about the subject matter.

In exercise section 1.15 I'm asked to prove a few increasingly
more difficult versions of the problem I posted here. Since
I use a similar strategy to prove each of those that I used to
prove the first, I figured that I should post that method and use
the responses as a litmus test to see if it was general considered
to be OK or generally considered to be NOT OK.

That's all I'm looking for, but as much as possible is welcome.

Thanks.

"Chairman of the Ozzy Osbourne Appreciation Society"
<mathgeek42@hotmail.com> wrote in message
news:84Cdna3KRe3Fm1LdRVn−hA@comcast.com...
> *Hi,*

>
> I'd like to get some comments on the following attempt
> at a proof, most importanly comments regarding any
> serious flaws and secondarily comments regarding
> ways to improve it.
>
> I'm asked to do the the following:
>
> If n is a postive integer, prove that:
>
> INT[floor(t)] dt from 0 to n = n(n−1)/2
>
>
> ======================= background =======================
>
>
> I'm given the definition of the integral from a to b of
> a step function as:
>
> INT[s(x)] dx from a to b = sum{k=1 to n}[S_k * (X_k − X_(k−1))]
>
> where P={X_0, X_1, X_2, ..., X_n} is a partition of [a,b]
> such that s is constant on the open subintervals of P.
>
> And Where S_k, is the constant value of s(x) in the kth open
> subinterval of P.
>
> =================== end of background =====================
>
> My thinking is to translate the integral into a sum, and
> then use the result of a previous excersize to show that
> the sum is equavalent to n(n−1)/2, but I'm uncertain if
> I'm going about this the right way.
>
>
> (1) INT[floor(t)] dt from 0 to n
>
> Let P = {0, 1, 2, ..., n} which satisfies the
> definitions above, from the definition of
> integral we have:
>
> (2a) 0 (1 − 0) + 1 (2 − 1) + 2 (3 − 2) + ... + (n−1) (n − (n−1))
>
> noting that the length of each open subinterval is 1,
> gives the following sum:
>
> (2) sum{k=0 to n−1} floor(k) * 1
>
> (3) As a consequence of the definition of floor, when x
> is an integer, x = floor(x). Since each k in the sum
> up to n−1 is an integer, we replace floor(k) with k.

> *Also since 1 is a constant multiplier, move it outside*
> *of the sum (and drop it), giving:*
>
> *(4) sum{k=0 to n−1} k*
>
> *(5*) since the 0th term is 0,*
>
> *sum{k=1 to n−1} k*
>
> *(6) From a previous result we know that*
>
> *sum{k=1 to n} k = n^2/2 + n/2*
>
> *(7) Subtracting the nth term we get a LHS that*
> *matches the sum in step 5*.*
>
> *sum{k=1 to n−1} k = n^2/2 + n/2 − n*
>
> *sum{k=1 to n−1} k = n^2/2 + n/2 −2n/2*
>
> *sum{k=1 to n−1} k = n^2/2 − n/2*
>
> *sum{k=1 to n−1} k = n (n−1)/2*
>
> *Which completes the proof.*
>
> *Thanks!*
>
>