

Re: A strange series

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From: Simeon Stefanov (*sim_stef_at_yahoo.com*)

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zellig@wanadoo.fr (georgesZ) wrote in message
news:<5227cf44.0406260837.2b67b03f@posting.google.com>...

> *sim_stef@yahoo.com* (Simeon Stefanov) wrote in message

>> *By the way, here is a continuous analogue:*

>> *Let $f(x)$ be a smooth function in $[1, \text{infinity})$ such that*

>> *i) $f(x) > 0$,*

>> *ii) $f(x) \rightarrow \text{infty}$, as $x \rightarrow \text{infty}$*

>> *iii) $|f'(x)| < M$.*

>> *Then the integral*

>> *Integral $[x \text{ from } 1 \text{ to } \text{infty}] [f(x) / F(x)]^2 dx$*

>> *is convergent, where*

>> *$F(x) = \text{Integral} [t \text{ from } 1 \text{ to } x] f(t) dt$.*

>

> *There is a problem for 1 (take $f(x)=x$).*

OK, the integral should be from 2 to infinity.

> *Could it be :*

> *Let $f(x)$ be a smooth function in $[1, \text{infinity})$ such that*

> *i) $f(1) > 0$,*

I suppose, you mean $f(x) > 0$, otherwise $F(x)$ may vanish and the
integral to diverge.

> *iii) $|f'(x)| < M$.*

> *Then the integral*

> *Integral $[x \text{ from } 2 \text{ to } \text{infty}] [f(x) / F(x)]^2 dx$*

> *is convergent, where*

> *$F(x) = \text{Integral} [t \text{ from } 1 \text{ to } x] f(t) dt$?*

It is possible that condition ii) $f(x) \rightarrow \text{infty}$ is unnecessary, I added
it by analogy with the original question.

>

> *Friendly,*

> *Georges*

Simeon