

Re: pain in neck calculus problem

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I have not solved this problem, but I sort of played with it a little, and here are some of my ideas.

Since we need 2 inflection points, I thought about looking at

$$y'' = (x - p)(x - q) = x^2 - (p + q)x + pq$$

Therefore, integrating once gives (with not constant term)

$$\begin{aligned} y' &= (x^3) / 3 - ((p + q) x^2) / 2 + pqx \\ &= (x / 6) (2x^2 - 3(p + q)x + 6pq) \end{aligned}$$

If my calculations are correct, the solutions to

$$2x^2 - 3(p + q)x + 6pq = 0$$

are

$$\left[\frac{3(p + q) \pm \sqrt{(3p - q)(p - 3q)}}{4} \right]$$

Then, in order to get integers, we at least need for

$$(3p - q)(p - 3q)$$

to be a perfect square. So, we can set up equations of the form

$$\begin{aligned} 3p - q &= k \\ p - 3q &= k \end{aligned}$$

I found that $k = 12$ will make integer solutions to the system of equations.

$$\begin{aligned} 3p - q &= 12 \\ p - 3q &= 12 \end{aligned}$$

However, that will not solve the original problem.

I think that going along these lines may work. You can let k be any multiple of 12 (I have no idea right now), or perhaps some other integer that makes

$$\left[3(p+q) \pm \sqrt{(3p-q)(p-3q)} \right] / 4$$

into at least one integer solution as well as have p and q be integer solutions.

Just my thoughts,

Brian

On Sun, 27 Jun 2004 03:14:22 GMT, "Troubled" <mkajumap@hotmail.com> wrote:

>I need to find a 4th degree polynomials with 3 extrema and 2 points of
>inflection, all 5 numbers being integers. I started off with letting $y' = x$
> $(x-p)(x-q)$, with p and q integers (so the critical values will be 0, p , and
> q). Then $y'' = (x-p)(x-q) + x(x-q) + x(x-p)$. I concluded that $y''=0$ when $[(p+q)$
> $\pm \sqrt{(p+q)^2 - 3pq}] / 3$. I know that $(p+q)^2 - 3pq$ must be a perfect
>square, I^2 . I concluded that $p = [q \pm \sqrt{(2I)^2 - 3q^2}] / 2$. This is
>where I got stuck. At this point I'd pick values for q and I and see if p is
>an integer and if it was I'd check to see if it satisfied the equation
>above. But I can't find a p, q combo that works, nor do I like to randomly
>pick numbers. Why is this problem so hard? Why can't I find a 4th degree
>polynomial with any 5 integers (or any 5 numbers) that satisfy the statement?
>Any hints or even solutions at this point would be appreciated. Believe it
>or not this is not homework. It is the problem of the month at a college
>which I do not attend that I came across.
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