

## Re: 2 rings with a special property

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On Tue, 29 Jun 2004, Bessel wrote:

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> *I want to find two rings  $R_1$ ,  $R_2$  and a homomorphism  $f: R_1 \rightarrow R_2$  between the two rings. I need some special properties:*

>

> *1.  $R_1$  should have many ideals*

> *2. Kernel of  $f$  should not look too "special" in any way.*

> *I.e. for example if we are dealing with matrices and the kernel of homomorphism is such that last column or last row is all zeroes, then it's not quite satisfactory because then it looks "special" as opposed to other regular elements which don't have this 0s property.*

> *3. I also would like  $|\ker f|/|R_1|$  to be fairly small.*

>

identity:  $\mathbb{Z} \rightarrow \mathbb{Z}$ .

$\mathbb{Z}$  has infinitely many ideals, kernel =  $\{0\}$ ,  $|\ker|/|\mathbb{Z}| = 0$

otherwise if division by  $|R_1|$  is clue  $R_1$  is finite

identity:  $\mathbb{Z}_n \rightarrow \mathbb{Z}_n$

$\mathbb{Z}_n$  has lots of ideals when  $n$  has lots of factors. Again kernel =  $\{0\}$  is most simple and  $|\ker|/|\mathbb{Z}_n| \rightarrow 0$  as  $n \rightarrow \infty$

> *Any suggestions of where to start?*

>

Plug the leaks in the problem statement?