Re: Rigorous proof of natural numbers' properties (by Edmund Landau).

Source: http://sci.tech−archive.net/Archive/sci.math/2004−07/0018.html

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From: Dan Christensen (*dchris_at_allstream.net*)
Date: 06/30/04

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Date: Wed, 30 Jun 2004 11:15:05 −0400
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correction

"ddtl" <this.is@invalid> wrote in message
news:svs4e0d5bcv7hgh2kv48jas94l786k8h9p@4ax.com...
>
> >*You are quite right. You can't just define addtion in this way. If you*
are
> >*starting from PA, you have to construct the add function from first*
> >*principles. First, you construct the set of ordered triples of natural*
> >*numbers, n3. Then you select a subset of n3. Then you must prove that*
this
> >*subset is a function with the required properties −− associativity, etc.*
The
> >*subset add can be defined as follows:*
>
> *I have to admit that I don't completely understand your explanation −*
probably
> *in order to do that one have to possess some knowledge in formal logic,*
and
> *my knowledge in mathematics is limited by high school math, but the*
following
> *have caught my attention:*
>
> *"First, you construct the set of ordered triples of natural numbers"*
>
> *There is a problem with that − in order to construct an ordered set of*
natural
> *numbers, you have to define what is "order" when applied to natural*
numbers,
> *and Landau defines "order" in terms of addition:*

The word "order" is used in different ways:

The set of ordered triples of natural numbers is just a set n3 such that for
all x,y, z, the ordered triple (x,y,z) is an element of n3 iff x, y and z
are are natural numbers. This construction does require one or more axioms

of set theory (depending on the system), but it does not require the use of addition.

The addition function (or any function of 2 variables) is itself a set of ordered triples of numbers. A function of 1 variable is a set of ordered pairs.

Consider a trivial example: f ={(x,x) | x is an element of set s}. You can easily prove that this set of ordered pairs is a function mapping n to itself (the identity function). Then you can write, for all x belonging to s, f(x) = x.

See my formal proof of this (generated using my DC Proof system) at

http://www.dcproof.com/IdentityFunction.html

Then of course, there is the (strict) ordering of the natural numbers which you define here.

> "Definition 2:
> If x=y+u, then x>y (> read: bigger)
>
> Definition 3:
> If y=x+v, then x<y (< read: smaller)"
>
> But if you cannot define and prove that addition exists, there is no
order,
> so it seems that the proof you are talking about is not valid.
>

Dan

Download DC Proof 1.0 at http://www.dcproof.com