

## Re: Rigorous proof of natural numbers' properties (by Edmund Landau).

*Source:* <http://sci.tech-archive.net/Archive/sci.math/2004-07/0054.html>

---

*From:* Leonard Blackburn ([blackbur\\_at\\_math.umn.edu](mailto:blackbur_at_math.umn.edu))

*Date:* 06/30/04

Date: 30 Jun 2004 10:06:46 -0700

David C. Ullrich <[ullrich@math.okstate.edu](mailto:ullrich@math.okstate.edu)> wrote in message  
news:<[u635e095i8kvh2gl6i2i6ig50lphv5gked@4ax.com](mailto:u635e095i8kvh2gl6i2i6ig50lphv5gked@4ax.com)>...  
> *On Wed, 30 Jun 2004 10:01:39 +0200, ddtl <[this.is@invalid](mailto:this.is@invalid)> wrote:*  
>

<snip>

>

> *Uh, that book has been around for a \_long\_ time (and Landau was*  
> *an extremely careful writer to begin with). If you think one*  
> *of the proofs is not valid you're missing something (perhaps*  
> *not entirely your fault, probably it would be written in*  
> *different language today).*

<snip>

Be careful Professor Ullrich. Landau's book does indeed contain a fatal error in the proof of the existence of an addition function. I admit I only briefly skimmed this thread, but I can't help diving in right now before reading it carefully, as this very issue arose for me two years ago. At the University of Minnesota, Professor Max Jodeit had been giving a course based in part on Landau's book (the one in question). He had been teaching the addition theorem as Landau had presented it for some time. I was his teaching assistant in 2002. I noticed the serious error in the proof right after having been doing a lot of logic and set theory including learning various recursion theorems (like the one in H. Enderton's book *\_A Mathematical Introduction to Logic\_*. I pointed out the error to Professor Jodeit, and he did not believe me for more than a month of debate. But in the end he came to complete agreement with me and he now teaches a correct proof based on a recursion theorem for the natural numbers as given in H. Enderton's *\_Elements of Set Theory\_*. In that book, Enderton even mentions that there are some erroneous proofs of the addition theorem in print. Landau's is one of them. Enderton mentions that if a proof does not utilize the Peano axioms that 0 is not in the range of the successor function and that the successor function is one-to-one, then the proof cannot be correct. See p. 76 in *\_Elements of Set Theory\_*.

sci.math: Re: Rigorous proof of natural numbers' properties (by Edmund Landau).

For my discussion of Landau's error go to

<http://www.math.umn.edu/~jodeit/course/Math3283S02.html>

which is Professor Jodeit's course page and click on this link near the bottom:

"Leonard Blackburn's Notes and Comments on the Addition Theorem."

In this document, I've written everything but the first paragraph (written by Professor Jodeit). I don't agree with the emphasis on the proof being ok except for missing justification that is expressed in that paragraph. The matter is also very briefly discussed in version 5 (?) of the Peano Postulates links.

I hope this document also helps the OP.

Sincerely,  
Leonard Blackburn