Re: better than a quartic?

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On 3 Jul 2004 07:17:07 −0700, Carlo_02@libero.it (Carlo) wrote:

> I have to solve this set of equations:
>
> t5 = t1 + (t1−t2)^2/((t1−t3)(Pa−1))*
>
> t1 = t5 + (t5−t4)^1/((t5−t3)(Pb−1))*
>
> t1 and t5 are unknown. t2, t3, t4, Pa, Pb are known.
> In practice, these properties are also true:
> 0 < t2 < t3 < t4
> 0 < Pa < 1
> 0 < Pb < 1
>
> With some manipulation, I can turn the equations above into a quartic,
> and solve it numerically (I know there is also a symbolic solutions to
> a quartic, however, the coefficients of this quartic are already a
> mess, never mind the symbolic quartic solution...). Once I get t1 or
> t5 this way, I can find the other easily by applying one of the above.
>
> Unfortunately, with the numbers I have to play with, the quartic is
> relatively a mess.

Probably the best that can be done algebraically is try and solve
for the unknowns in terms of a parameter that satisfies a simpler
quartic than maybe the one you found.

For typing convenience I've relabelled your variables and unknowns
as follows:

x, y, a, b, c, d, e := t5, t1, t2, t4, Pa − 1, Pb − 1, t3

so your equations become:

x − y = (x − b)^2 / (d.(x − e)) = (y − a)^2 / (c.(y − e))

taking:

  u, v, A, B := (x − b)/d, (y − a)/c, (b − e)/d, (a − e)/d

these become:

  d.u − c.v + b − a = u^2 / (u + A) = v^2 / (v + B)

The latter pair are equivalent to:

  B.((2A/u + 1)^2 − 1) = A.((2B/v + 1)^2 − 1)

So taking:

  2A/u, 2B/v = s, s.t

gives either s = 0 or a linear equation in s which implies
the following general parametrization of u, v in terms of t:

  u = t.v = −A.(A.t^2 − B)/(A.t − B)

and plugging that into "d.u − c.v ... = " gives the quartic:

  (A.t^2 − B)^2 + (A.c.t − B.d + b − a).(t − 1).(A.t^2 − B)

        − B.(b − a).(t − 1)^2 = 0

This is a homogenous quadratic in A.t^2 − B and t − 1.
But unfortunately if A.c != 0 there is a pesky t in the
middle coefficient, and this means it can't be solved
directly as a pair of quadratic steps.

(If you decide to use the preceding, it would be a good idea
to check the algebra!

Cheers

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"Eternity is a long time, especially towards the end."
  Woody Allen