

Re: Confused with uniform/pointwise convergence

Source: <http://sci.tech-archive.net/Archive/sci.math/2004-07/3221.html>

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Date: 07/13/04

Date: Tue, 13 Jul 2004 09:31:20 -0500

On Tue, 13 Jul 2004 09:50:57 -0400, "Isaac" <isharu@yahoo.com> wrote:

>
>[...]
>
>Dear David,
>
>Thanks for your responses! Last question : I was wondering, because it
>doesn't say in my text,
>are there less restrictive requirements to arrive at the "conclusion"?

Perhaps...

>Because like I mentioned before,
>it seems to me that the "conclusion" is the very important (i.e. when can we
>integrate term
>by term?) part/consequence of this theorem. So, maybe if we relax the
>assumptions of the theorem a bit, like $f_n \rightarrow f$ pointwise, then we can
>still have $g_n \rightarrow g$ pointwise (which is the same as the "conclusion"), but
>not uniformly.

No, absolutely not! Negatory. No way.

Your first question was very easy – the answer was yes, just from the definition of g_n and g . The fact that the question you asked was *_so_* easy is I suspect part of the reason why people read the question wrong and said no. If you look at the replies that said no you'll find that *_this_* question is what they were giving counterexamples for (at least i think so, didn't read them very carefully).

>Because now if someone asks me when can you integrate
>a sequence term by term, I will say that the sequence must be riemann
>integrable and uniformly convergent, but these aren't necessary conditions
>it seems to me, they are sufficient. Well actually, I believe that f_n must
>be riemann integrable, I believe that has to be necessary. But do f_n have
>to converge uniformly to f ? Maybe a less restrictive convergence?

Uniform convergence is more or less the best condition that springs to mind when we're just talking about the Riemann integral. You will find much weaker conditions that allow you to say the integral of the limit is the limit of the integral later on, when you study the Lebesgue integral (in "real analysis" or "measure theory".)

>*Thanks,*

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>*Isaac*

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David C. Ullrich