

Re: Peano's space-filling curve

Source: <http://sci.tech-archive.net/Archive/sci.math/2004-07/3311.html>

From: John Morgan (*john.morgan_at_atExpungebetweencapSaraxie.fr*)

Date: 07/13/04

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I've replied to four posts in one to save space and time. No Lorentz transformation required :-))

David C. Ullrich <ullrich@math.okstate.edu> wrote in message news:j5bqe095dkosep86mj40e3ef170kqek1ma@4ax.com...

> On Thu, 8 Jul 2004 12:32:13 +0200, "John Morgan"

> <john.morgan_at_atExpungebetweencapSaraxie.fr> wrote:

>

> > I meant $A = \{a_1, a_2, a_3, \dots\}$ and similarly for A' , B and

> > B' , where a_1, a_2, \dots etc. are the individual elements.

>

> Not sure exactly what you mean by that. The notation

> appears to suggest that A and B are countable, which

> they need not be.

I see from Dan Grubb's post that the correct notation should be $A = \{a: a \in A\}$

[...]

> A is a set; that means it has some things that we

> call elements, some > things are elements of A

> and some things are not. That's all you need to know.

I can conceive of this, but then I encounter a new problem. It always seemed to me that the domain of a function and the rule that linked it to the codomain were inextricably linked, with the set comprising the domain determining which rules were applicable, while a given rule might only be relevant to a certain class(es) of sets. If the domain was a set of the names of famous people, then the rule 'has a birthday on' can map the name to a codomain of 366 Julian days, while the rule 'multiply by two' is meaningless. However, its inverse 'multiply by 1/2' is not incompatible with the codomain. Similarly the first mentioned rule is meaningless when applied to a domain of integers while its inverse 'is the birthday of' can be applied to integers, as long as these represent Julian days.

What I see now is a level of abstraction wherein the nature of the function is made sufficiently general that any and all sets can be regarded as its domain/codomain. I probably need to think about this quite a bit.

> >[...]

> >if I take TF1 as true then I have no other problems
> >with following the proof and finding it makes sense –
> >which is all that this scientist needs :-)
>
> It's hard for me to see how you could do the rest of it
> if you're unable to do TF1.

I just_assumed_TF1 was correct and continued just as though I had_proved_it correct. See below.

> *But there are lots of things I don't understand.*

I know the feeling :-))

Daniel Grubb <grubb@lola.math.niu.edu> wrote in message news:ccjmrp\$it9\$1@news.math.niu.edu...

>
> *Questions involving functions between sets*
> *only involve the sets *as sets*. If there are orders on*
> *the , we can also talk about 'order preserving functions'*
> *and if there are topologies on the sets we can talk about*
> *'continuous functions', but a general function doesn't*
> *have to be either of those.*

[...]

> $A \times B = \{(x,y): x \in A, \text{ and } y \in B\}$.
>
> *Still good?*

A OK

> *Now, for $\mathbb{R} \times \mathbb{R}$, we usually imagine the plane, since*
> *Descartes showed how to identify the plane with*
> *ordered pairs of real numbers. But, we don't have a*
> *notion of closeness when we are dealing with sets.*
> *Each point of the plane is regarded as separate from*
> *every other. No closer or farther away.*

I have difficulty with 'separate' applied to ordered reals. That would seem to me to invest them with the property of countability which I know they don't have. I seem to see in Cantor's diagonal argument that no amount of separation would ever, ultimately, allow the reals to be listed in

order and that they are therefore inseparable.

>

David C. Ullrich <ullrich@math.okstate.edu> wrote in message
news:1m5re0pu82r8r7qnlkckvpcgkinp1h12pn@4ax.com...

> *On 8 Jul 2004 14:48:25 GMT, grubb@lola.math.niu.edu*
(Daniel Grubb)

> *Under the circumstances it probably would be better not*
> *to corrupt the notation.*

Not to worry, I was able to read beyond it.

I had more or less got to $F((a,b)) = (f(a),g(b)) = (a',b')$
because there was really nowhere else to go! I was still
unhappy that this ignored what elements were in A and B and
also what kind of functions F, f and g were. I'm probably
too old for this kind of mathematics :-(

> >...*need more than the hints I was giving.*

> >...*he had got it himself he would have attained*

> >*a deeper understanding.*

In fact I seem to be striving for a depth of understanding
that is not actually required :-(

Daniel Grubb <grubb@lola.math.niu.edu> wrote in message
news:cck69q\$kfpl@news.math.niu.edu...

> *I thought a nudge would do him good. He still has to show*
> *that the new function is a one-to-one correspondence, so*
> *there's plenty of chances to learn. :)*

I think it is because $f^{-1}(a') = a$ and $g^{-1}(b') = b$ then
 $F^{-1}((a',b')) = (f^{-1}(a'),g^{-1}(b')) = (a,b)$ so the existence of
an inverse function implies a 1-to-1 correspondence for F.

For the rest of the proof of TF4 $I \times I = I$, I simply noted
that TF1 $A \times B \sim A' \times B'$ taken with TF3 $D \sim I$ gives TF5 $D \times D \sim I$
 $\times I$, TF2 $D \times D \sim D$ taken with TF3 $D \sim I$ gives TF6 $D \times D \sim I$ and
these two new TFs together give the desired result $I \times I \sim I$

Cheers,

John