

## Re: Peano's space-filling curve

**Source:** <http://sci.tech-archive.net/Archive/sci.math/2004-07/3393.html>

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**From:** David C. Ullrich ([ullrich\\_at\\_math.okstate.edu](mailto:ullrich_at_math.okstate.edu))

**Date:** 07/13/04

Date: Tue, 13 Jul 2004 18:09:25 -0500

On Tue, 13 Jul 2004 17:39:46 +0200, "John Morgan"  
<[john.morgan@atExpungebetweencapSaraxie.fr](mailto:john.morgan@atExpungebetweencapSaraxie.fr)> wrote:

>I've replied to four posts in one to save space and time. No  
>Lorentz transformation required :-))

>

>David C. Ullrich <[ullrich@math.okstate.edu](mailto:ullrich@math.okstate.edu)> wrote in message  
>news:j5bqe095dkosep86mj40e3ef170kqek1ma@4ax.com...

>> On Thu, 8 Jul 2004 12:32:13 +0200, "John Morgan"

>> <[john.morgan@atExpungebetweencapSaraxie.fr](mailto:john.morgan@atExpungebetweencapSaraxie.fr)> wrote:

>>

>> >I meant  $A = \{a_1, a_2, a_3, \dots\}$  and similarly for  $A'$ ,  $B$  and

>>  $B'$ , where  $a_1, a_2, \dots$  etc. are the individual elements.

>>

>> Not sure exactly what you mean by that. The notation

>> appears to suggest that  $A$  and  $B$  are countable, which

>> they need not be.

>

>I see from Dan Grubb's post that the correct notation should

>be  $A = \{a: a \in A\}$

Hmm. That doesn't really say anything at all;  $\{a: a \in A\}$   
is a synonym for  $A$ .

> [...]

>

>>  $A$  is a set; that means it has some things that we

>> call elements, some > things are elements of  $A$

>> and some things are not. That's all you need to know.

>

>I can conceive of this, but then I encounter a new problem.

>It always seemed to me that the domain of a function and

>the rule that linked it to the codomain were inextricably

>linked, with the set comprising the domain determining which

>rules were applicable, while a given rule might only be

>relevant to a certain class(es) of sets. If the domain was a

>set of the names of famous people, then the rule 'has a

>birthday on' can map the name to a codomain of 366 Julian

>days, while the rule 'multiply by two' is meaningless.  
>However, its inverse 'multiply by 1/2' is not incompatible  
>with the codomain. Similarly the first mentioned rule is  
>meaningless when applied to a domain of integers while its  
>inverse 'is the birthday of' can be applied to integers, as  
>long as these represent Julian days.  
>  
>What I see now is a level of abstraction wherein the nature  
>of the function is made sufficiently general that any and  
>all sets can be regarded as its domain/codomain. I  
>probably need to think about this quite a bit.

It's certainly true that a given rule can only apply to certain classes of sets, etc. But here we don't need to know anything about what the functions  $f$  and  $g$  actually *are*, we're *given* that they are functions with certain domains, etc. That's all we know about them, and all that we need to know.

>> >[...]  
>  
>> >if I take TF1 as true then I have no other problems  
>> >with following the proof and finding it makes sense –  
>> >which is all that this scientist needs :-)  
>>  
>> It's hard for me to see how you could do the rest of it  
>> if you're unable to do TF1.  
>  
>I just *assumed* TF1 was correct and continued just as though  
>I had *proved* it correct. See below.

I knew that was what you said you'd done – I found it hard to believe that if you were having trouble with TF1 you'd be able to do the others, since they all seemed to me to be about the same difficulty. But I just looked, and come to think of it I suppose it could be that TF1 is more abstract than the others, so I suppose it's believable.

>> *But there are lots of things I don't understand.*  
>  
>I know the feeling :-))  
>  
>Daniel Grubb <grubb@lola.math.niu.edu> wrote in message  
>news:ccjmrp\$it9\$I@news.math.niu.edu...  
>>  
>> *Questions involving functions between sets*  
>> *only involve the sets \*as sets\*. If there are orders on*  
>> *the , we can also talk about 'order preserving functions'*  
>> *and if there are topologies on the sets we can talk about*  
>> *'continuous functions', but a general function doesn't*

>> *have to be either of those.*  
>  
>[...]  
>  
>>  $A \times B = \{(x,y): x \text{ in } A, \text{ and } y \text{ in } B\}$ .  
>>  
>> *Still good?*  
>  
>A OK  
>  
>> *Now, for  $\mathbb{R} \times \mathbb{R}$ , we usually imagine the plane, since*  
>> *Descartes showed how to identify the plane with*  
>> *ordered pairs of real numbers. But, we don't have a*  
>> *notion of closeness when we are dealing with sets.*  
>> *Each point of the plane is regarded as separate from*  
>> *every other. No closer or farther away.*  
>  
>*I have difficulty with 'separate' applied to ordered reals.*  
>*That would seem to me to invest them with the property of*  
>*countability which I know they don't have. I seem to see in*  
>*Cantor's diagonal argument that no amount of separation*  
>*would ever, ultimately, allow the reals to be listed in*  
>*order and that they are therefore inseparable.*

Here I think you're thinking too much about what Grubb said. All he meant is that here we're totally ignoring almost all the aspects of the real numbers that are usually what makes them interesting. The fact that you can add two reals and get another one is irrelevant. The fact that they're lined up on that line is irrelevant; the fact that these two are close to each other and those two are far from each other is irrelevant.

Here the reals are just a pure set, not a set-plus-way-to-add-two-elements or a set-plus-notion-of-the-distance-between-two-elements or any such thing.

>*David C. Ullrich <ullrich@math.okstate.edu> wrote in message*  
>*news:1m5re0pu82r8r7qnlkckvpcgkinp1h12pn@4ax.com...*  
>> *On 8 Jul 2004 14:48:25 GMT, grubb@lola.math.niu.edu*  
>*(Daniel Grubb)*  
>  
>> *Under the circumstances it probably would be better not*  
>> *to corrupt the notation.*  
>  
>*Not to worry, I was able to read beyond it.*  
>  
>*I had more or less got to  $F((a,b)) = (f(a),g(b)) = (a',b')$*   
>*because there was really nowhere else to go! I was still*  
>*unhappy that this ignored what elements were in A and B and*

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>also what kind of functions  $F$ ,  $f$  and  $g$  were. I'm probably  
>too old for this kind of mathematics :-(  
>  
>> >....need more than the hints I was giving.  
>  
>> >....he had got it himself he would have attained  
>> >a deeper understanding.  
>  
>In fact I seem to be striving for a depth of understanding  
>that is not actually required :-(

Heh. Yes, that could be. You say you can't conceive of this  
or that – \_don't try\_, it doesn't matter!

>Daniel Grubb <grubb@lola.math.niu.edu> wrote in message  
>news:cck69q\$kfj\$1@news.math.niu.edu...  
>  
>> I thought a nudge would do him good. He still has to show  
>> that the new function is a one-to-one correspondence, so  
>> there's plenty of chances to learn. :)  
>  
>I think it is because  $f^{-1}(a') = a$  and  $g^{-1}(b') = b$  then  
> $F^{-1}((a',b')) = (f^{-1}(a'),g^{-1}(b')) = (a,b)$  so the existence of  
>an inverse function implies a 1-to-1 correspondence for  $F$ .  
>  
>For the rest of the proof of TF4  $I \times I = I$ , I simply noted  
>that TF1  $A \times B \sim A' \times B'$  taken with TF3  $D \sim I$  gives TF5  $D \times D \sim I$   
> $\times I$ , TF2  $D \times D \sim D$  taken with TF3  $D \sim I$  gives TF6  $D \times D \sim I$  and  
>these two new TFs together give the desired result  $I \times I \sim I$   
>  
>Cheers,  
>  
>John  
>  
>  
>  
>  
>  
>  
>  
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>  
>

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David C. Ullrich