Re: normal subgroup

Source: http://sci.tech−archive.net/Archive/sci.math/2004−07/5231.html

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Date: 07/20/04

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Date: Tue, 20 Jul 2004 15:30:03 +0000 (UTC)
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In article <cdj6lajvg1@news−reader2.wanadoo.fr>,
"Julien Santini" <santini.julien@wanadoo.fr> writes:
>> let G be a finite group . let H be asubgroup of G and let N be a normal
>subgroup of G .prove that if |H| and |G:N| are relatively prime . Then H is
>a subgroup of N .
>>
>
>|N| = sum(p_i^k_i, i = 1..n) where n is a positive integer, p_i are distinct
>prime numbers and k_i are positive integers. (the case |N|=1 being trivial)
>For each i, there is only a single p_i−Sylow subgroup in N (because N is
>normal, whence each of its Sylow subgroups is normal too).

That does not follow! According to that argument, every Sylow subgroup of
every finite group G is normal − just take N = G.

Derek Holt.

>Now take an element h in H. We know that its order is of the form:
>sum(p_i^o_i, i =1..n) where o_i are nonnegative integers. Also, the
>generated subgroup <h> is cyclic, hence abelian, and the direct sum of its
>Sylow subgroups. But any p_i−group of G must be contained in a p_i−Sylow of
>G, and the p_i−Sylows are uniquely determined, whence the p_i−Sylows of <h>
>are contained in those of N, and <h> is contained in N. In particular, h is
>in N.
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>−−
>Julien Santini
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