Re: Distribution of random sum of random variables

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In article <a07a8e30.0407270736.1277df26@posting.google.com>,
Eric Wong <wongman_eric@yahoo.com> wrote:
>Suppose that
>1. N is a discrete nonnegative random variable
>2. X_i's are i.i.d. random variables
>2. N and X_i are independent

>Is it true that when E[N] tends to infinity, the distribution of the
>following random sum converges to the normal distribution? (as I guess
>it may be an extension of the Central Limit Theorem.)

>S = sum(i=1..N) X_i

I'm guessing that what you mean is something like this:
Let the distribution of N depend on some parameter t, such that E[N] = t.
As t −> infinity, does (S − E[S])/sqrt(Var(S)) converge in distribution
to a normal distribution?

The answer is no in general. Take, for example, a case where
N = 0 with probability 1/2 and 2t with probability 1/2.

>Furthermore, for the special case that N and X_i are both Poisson, is
>there some well−known distribution that describes S?

The phrase to look up is "compound Poisson process", I think.

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