Internal Set Theory Uniqueness Principle

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In Edward Nelson's original paper on Internal Set Theory, he gave a proof that if

(1) P(x) is a formula in Internal Set Theory with one free variable x,

(2) P(x) is relativized to a standard set V (i.e. all quantifiers are of the form "for all x in V", "for some x in V", "for all standard x in V", "for some standard x in V",

(3) there is a unique value of x in V for which P(x) holds,

then the unique value of x in V, such that P(x) holds, is standard.

My question is that if

(1) P(u,v_1,...,v_k) is a formula in Internal Set Theory,

(2) y_1, ..., y_k are standard,

(3) there exists exactly one x such that P(x,y_1,..,y_k) holds,

then is it necessarily true that the unique value of x specified in (3) is standard?

Nelson answered the question in the case where there is only one free variable and the formula is relativized to a standard set V, with the answer being in the affirmative. Nelson's proof is easily extendible to the case where there is more than one free variable. I am curious about the more general case (i.e. when the formula is not relativized to a standard set).

If there is a proof in the general case that x must be standard, I would be grateful to see it. If it is not true in general that x must be standard, please give an example. Thanks.

David

"But I'm always true to you, darlin', in my fashion,
Yes, I'm always true to you, darlin', in my way."
−− Lois Lane

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