Re: Reference for a cubic with a double root?

Source: http://sci.tech−archive.net/Archive/sci.math/2004−08/2118.html

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Date: 08/11/04

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Date: 11 Aug 2004 02:42:57 −0400
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Many thanks to all who responded! In a way my question is still unanswered (see end of my post), but let me first point out that there are at least three ways of approaching this problem:

(1) The 'standard' one, self−evident to those who do remember their Algebra and the fact that the discriminant of the cubic ax^3 + bx^2 + cx + d = = a(x−X1)(x−X2)(x−X3) , that is [(X1−X2)(X2−X3)(X3−X1)]^2, is equal to (b^2)(c^2) − 4a(c^3) − 4(b^3)d − 27(a^2)(d^2) + 18abcd: there is a double root iff (b^2)(c^2) − 4a(c^3) − 4(b^3)d − 27(a^2)(d^2) + 18abcd = 0. [This was suggested by Ken Pledger.]

(2) An approach making 'limited use' of the derivative 3ax^2 + 2bx + c in order to cleverly arrive at (2b^3 − 9abc + 27(a^2)d)^2 = 4(b^2 − 3ac)^3, a condition easily seen to be equivalent to the one above. [This was suggested by Rob Johnson, and, in the case b = 0, by Michael Jorgensen.]

(3) A 'Calculus' approach according to which there will be a double root iff one of the derivative's root(s), that is −b/3a +− sqrt(b^2−3ac)/3a, is also a root of the cubic: substituting into the cubic, we see that all the radicals are eliminated −− regardless of whether it's the + or − sign that is in effect −− and that we obtain a value of zero for the cubic precisely when (2b^3 − 9abc + 27(a^2)d)^2 = 4(b^2 − 3ac)^3. [This is 'my' way.I

So, what do I need now? Well, I still need a reference where the condition is literally spelled out, *not for the special case b = 0*, but for the general case, *preferably even when a is not taken to be equal to 1* :−) [I see that this is done in (my Aristotle University professor) Konstantinos Lakkis's "Algebra" (1976), but that book is written in Greek, and I need a reference in English; C. C. MacDuffee's "Theory of Equations" (1954) comes close to what I need in Theorem 50* (p. 91), except that he assumes a = 1 (a very minor offense which I would still prefer to avoid if possible).]

*With D = (b^2)(c^2) − 4(c^3) − 4(b^3)d − 27(d^2) + 18bcd, the cubic x^3 + bx^2 + cx + d has a double (real) root in case D = 0, three distinct real roots in case D > 0, and precisely one real root in case D < 0 : there is a reason that D is called "discriminant", in case you forgot :−)

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One possible reason for the creature's sudden fit of fury may have been an unconfirmed report that it was "kicked by somebody in business class" on its way through the cabin [http://news.bbc.co.uk/2/hi/europe/3551672.stm]