

Re: Rado's Sigma and the Halting Problem for Programs

Source: <http://sci.tech-archive.net/Archive/sci.math/2004-08/2460.html>

From: peter_douglass (*baisly_at_gis.net*)

Date: 08/12/04

Date: Thu, 12 Aug 2004 23:22:59 GMT

"r.e.s." <r.s@ZZmindspring.com> wrote in message
news:%HASc.18697\$cK.14152@newsread2.news.pas.earthlink.net...

> "peter_douglass" <baisly@gis.net> wrote ...

> -snip-

> > I believe the Halting Theorem may be proven without recourse

> > to *reductio ad absurdum*. Here is my stab at it.

> -snip-

> > Assume that from a Turing Machine M we can construct

> > ^^^^^

> > a Turing Machine M' such that the following holds:

> > $Eval(M',x) == begin$

> > if $Eval(M, \langle G'(M'),x \rangle) == "no"$

> > then return "yes"

> > else $loop_forever()$;

> > end

> -snip-

> Can you prove, without *reductio ad absurdum*, that such

> an M' can always be constructed?

Hmm. I'm not sure. I think you have pointed out a bug
in the proof. Better would be

```
Eval(M',x) == begin
    if Eval(M,<x,x>) == "no"
        then return "yes"
        else loop_forever() ;
end
```

This change does not affect the form of the proof
because when evaluating $Eval(M',G'(M'))$
the body is still instantiated as

begin

if Eval(M,<G'(M'),G'(M')> == "no"

etc.

- > *The self-referring definition of M' would seem to make*
- > *that less tractable than the usual type of definition of*
- > *a "derived TM" for reductio ad absurdum, e.g. (using the*
- > *usual shorthand suppressing the encoding of TMs & inputs):*
- > *M'(P) converges iff M(P,P) converges to "no".*
- > *Defining M' in terms of M alone is tractable, but your*
- > *version may not be. (?)*

Good observation. Hope there is nothing so wrong with the "corrected" version.

—PeterD