Re: [Collatz] was : Re: Status of Waring−problem − Collatz − Sorry

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From: Mensanator (*mensanator_at_aol.compost*)
Date: 08/28/04

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Date: 28 Aug 2004 16:47:41 GMT
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>Subject: Re: [Collatz] was : Re: Status of Waring−problem − Collatz − Sorry
>From: Gottfried Helms helms@uni−kassel.de
>Date: 8/28/2004 3:14 AM Central Standard Time
>Message−id: <cgpf55$9t0$05$1@news.t−online.com>
>
>Am 28.08.04 06:48 schrieb Mensanator:
>>>
>>> 3^N − 2^N
>>> a = −−−−−−−−−−−−−− (condition for a 1−cycle or "primitve loop")
>>> 2^S − 3^N
>>>
>>>The solution is only valid if a is odd and is integer. For N=1 and A=2 we
>get
>>>the trivial loop of one step:
>>> 1
>>> a = C(a;2) = −−− = 1
>>> 1
>>>
>>
>>
>> Does this only apply to positive integers? The reason I ask is my
>formulations
>> span both positive and negative domains and I get two 1−cycle loops:
>>
>> 1 −> 4 −> 2 −> 1
>>
>> and
>>
>> −1 −> −2 −> −1
>>
>Well, in my notation this would be
>
> 2^1
> x = C(x;1) = x* −−− + C(0;1)
> 3^1

>
> 3^1 3^1 − 2^1 3^1
> x = C(0; 1) * −−−−−−−−−− = −−−−−−−−−−−−* −−−−−−−− = −1
> 2^1 − 3^1 3^1 2^1 − 3^1
>
>>From the notation there is no specific restriction on numbers x.
>You even can use rationals, if you do some more finetuned analysis.
>All these are just compacted notations, no new inventions.

Ok, I just wanted to be clear on that. Obviously, all correct algorithms must reach the same conclusion. In my system, the transform structure creates a function that is a straight line. This line cannot have a slope of 1, so it must intersect the line y=x. If the intersection is an integer, then you have a loop. The intersection can be positive or negative, it all depends on the transform structure.

>From these structures, there are four loop points that are found in
all 3x+C systems: +C, −C, −5C, −17C. All lot of people seem to consider the positive and negative domains to be seperate, but they are tied together when you consider transform structures.

>
>−−−−−−−−−−−−−−−−−−−−−−−−−−−−
>
>For instance, another thing that you immediately see −if you use this
>notation−
>is, that infinetly many solutions in x' and x exist for a certain
>transformation−
>structure, say

In my system, the function is

A' = (X*A − Z)/Y

where A' is the result of the transform from A (this is going up the tree using x*2 and (x−1)/3 rules). Once you find the first A that gives an integer A', you can generate an infinite number of solutions by adding multiples of Y to A.

To find A for any given transform, the function can be converted to a problem in linear congruence:

X*A == Z (mod Y)

which is solvable if GCD(X,Y) divides Z. SInce X is a power of 2 and Y is a power of 3, the GCD will always be 1, so the problem always has a solution.

So not only does each solvable transform exist infinitely many times, _every_ possible legal transform exists somewhere on the Collatz tree. I don't know if that means anything, but it's interesting.

>
> x' = C(x; A,B,C)
>
>This is
> 2^(A+B+C)
> x' = x * −−−−−−−−−− + C(0;A,B,C)
> 3^3
>
>The C(0;...)− part is in general a fraction with the denominator 3^N, in this
>case of a three−step−transformation it is 3^3.
>So you see, that for all x with the same modular−class based on 3^3 you
>find an integral solution in x and x' for the transformation x−>x' , or
>can try to find approriate exponents for a given pair (x,x'), for instance
>x' = 2x+1 or x'−x = 2^B or anything the like.
>
>Extended to the question of cycles it is simply demonstrable, that
>for a certain sequence of exponents only one solution for x is possible;
>that means, a search for possible loops over reduces to a search for
>configurations of exponents, and strong restrictions can be stated for
>such a sequence: the sum S of all exponents must between about 1.5*N and
>2*N and the like.
>
>−−−−−−−−−−−−−−−−−−−−−−−−−−−−−
>
>Using a certain sequence of exponents A,B,C,
>
> x' = C(x; 2,2,1,2,1,...)
>you can construct numbers x−>x' where x has an arbitray long trajectory ,
>where the intermediate values never fall below x (I think, this is called
>"glide" ?).

That's interesting. One of the things I was looking at is why Mersenne numbers, which have the highest excursion (largest value in sequence) don't ever seem to be sequence length record holders. Part of this seems to be related to the glide. The slope of the line up to the excursion is steep, but so is the fall from the excursion. The record holders have a gentler slope up to a lesser excursion, but also a gentler slope down from the excursion producing a longer glide. I haven't had much luck constucting sequence length record holders, so maybe the secret is focusing on the glide.

>There are arbitrarily many transformations x' = C(x; A,B,C..), but to
>be applied to a pair of integral (x,x') it seems to me, that this sequence
>allows the smallest numbers (empirically and only few tests, maybe there
>are easy counterexamples except the trivial one).
>
>−−−−−−−−−−−−−−−−−−−−−−−−−−−−−−−−
>
>and so on.
>As I said, I suggest it as a practical notation for the whole
>collatz−calculus.

Now I just have to try to understand it.

>
>*Gottfried Helms*

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Mensanator
Ace of Clubs
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