

## Re: [Collatz] was : Re: Status of Waring-problem – Collatz – Sorry

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**From:** Gottfried Helms ([helms\\_at\\_uni-kassel.de](mailto:helms_at_uni-kassel.de))

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Date: Sat, 28 Aug 2004 23:56:28 +0200

Am 28.08.04 18:47 schrieb Mensanator:

[big snip, being a bit tired at this evening]

>

> *That's interesting. One of the things I was looking at is why Mersenne numbers, which have the highest excursion (largest value in sequence) don't ever seem to be sequence length record holders. Part of this seems to be related to the glide. The slope of the line up to the excursion is steep, but so is the fall from the excursion. The record holders have a gentler slope up to a lesser excursion, but also a gentler slope down from the excursion producing a longer glide. I haven't had much luck constructing sequence length record holders, so maybe the secret is focusing on the glide.*

>

>

Mersenne numbers are that of the form  $2^N - 1$ , right?

If you look at series of trajectories of some transformation of the same length, say

```
writetrajectory(findfirst(A,B,C,D))
```

modifying the exponents A,B,C,D, then there are two important observations:

```
==> for i:=1 to 8 do
```

```
  writeln({i,1,1},"", trajectory({i,1,1}));
```

```
end;
```

```
.
```

Exponents: ! lowest trajectory

```
T(a;A,B,C) !
```

```
-----!-----
```

```
(1, 1, 1) :(15, 23, 35, 53)
```

```
(2, 1, 1) :(9, 7, 11, 17)
```

```
(3, 1, 1) :(61, 23, 35, 53)
```

```
(4, 1, 1) :(37, 7, 11, 17)
```

```
(5, 1, 1) :(245, 23, 35, 53)
```

```
(6, 1, 1) :(149, 7, 11, 17)
```

```
(7, 1, 1) :(981, 23, 35, 53)
```

```
(8, 1, 1) :(597, 7, 11, 17)
```

here we see, that the outcoming is only 53 or 17 ( $53=2*3^3-1$  and  $17=2*3^2-1$ ) depending on the remainder of the first exponent modulo 2. Note, that only the trajectory for  $T(a;1,1,1)$  requires a first member "a" of the mersenne–form  $2*2^3-1 = 15$ .

Now we change the second exponent:

```
==> for i:=1 to 8 do
  writeln({1,i,1},":", trajectory({1,i,1}));
end;
.
Exponents: ! lowest trajectory
T(a;A,B,C) !
-----!-----
(1, 1, 1):(15, 23, 35, 53)
(1, 2, 1):(27, 41, 31, 47)
(1, 3, 1):(19, 29, 11, 17)
(1, 4, 1):(67, 101, 19, 29)
(1, 5, 1):(163, 245, 23, 35)
(1, 6, 1):(99, 149, 7, 11)
-----
(1, 7, 1):(995, 1493, 35, 53) * repeating modulo class 53
(1, 8, 1):(1763, 2645, 31, 47)
```

Now we have six different results as endpoint of the complete trajectory, depending on the remainder of the second exponent modulo 6. Also we can see, that they all are below 54

Now changing 3'rd exponent:

```
==> for i:=1 to 21 do
  writeln({1,1,i},":", trajectory({1,1,i}));
end;
.
Exponents: ! lowest trajectory
T(a;A,B,C) !
-----!-----
(1, 1, 1):(15, 23, 35, 53)
(1, 1, 2):(7, 11, 17, 13)
(1, 1, 3):(55, 83, 125, 47)
(1, 1, 4):(87, 131, 197, 37)
(1, 1, 5):(23, 35, 53, 5)
(1, 1, 6):(407, 611, 917, 43)
(1, 1, 7):(663, 995, 1493, 35)
(1, 1, 8):(1175, 1763, 2645, 31)
[...]
(1, 1, 18):(970903, 1456355, 2184533, 25)
-----
(1, 1, 19):(4116631, 6174947, 9262421, 53) * repeating modulo class 53
(1, 1, 20):(2019479, 3029219, 4543829, 13)
(1, 1, 21):(14602391, 21903587, 32855381, 47)
```

we get all residue classes  $6k+1, 6k-1$  of 54, which is  $2*3^3$  and gives 18 residue–classes that way. The important observation is, that the mersenne–requiring combination of exponents (always  $T(a;1,1,1,1,\dots,1)$ ) also gives the highest value of the residue–classes.

These "mersenne"–trajectories are the same as I call them "primitive transformation" (or "primitive loop" if a descendent step is added) and transform from

$$2*2^{(N-1)} - 1 \longrightarrow 2*3^{(N-1)} - 1$$

where the rhs is the highest value in the residue class of  $2*3^{(N-1)}$

\*All\* other transformations  $T(a;A,B,C)$  land in a lower class than  $T(a;1,1,1)$ . One could state that as a theorem. Now we have lots of other combinations of exponents of the same length; some are keeping the following members constantly above the starting value:

$T(a;1,2,1)$  for instance gives

Exponents: ! lowest trajectory  
 $T(a;A,B,C)$  !  
-----!  
(1, 2, 1):(27, 41, 31, 47)

So it is easy to construct trajectories, which do not fall below their initial value, and this need not explicitly be proven.

Since  $T(a;1,1,1)$  lands on the highest residue–class of  $2*3^3$  all other transformations of the same length land on lower classes, but since we can always construct trajectories, which as well do not fall below their initial value, we can conclude, that we always can construct a glide of the same length as the mersenne–trajectory but with a lower ending point.

(hope I didn't miss your point, possibly it's getting too late)

good night –

Gottfried Helms