Re: Uncountable sets in CZF?

Source: http://sci.tech−archive.net/Archive/sci.math/2004−08/5421.html

---

From: KRamsay (*kramsay_at_aol.com*)
Date: 08/30/04

```
Date: 30 Aug 2004 08:58:14 GMT
```

raf@tiki−lounge.com (Ross A. Finlayson) writes:
|Now we've seen in other threads that in IZF it is not inconsistent for
|there to be bijections between N and R. Is that not so?

No, Cantor's theorem that for any function from N to R there exists an element of R not in the image is a theorem of IZF. Please just think of this as a mathematical fact, won't you?

I've seen proofs that it's consistent with various constructive formal systems to assume that each function from N to N is computable. If I'm not mistaken the same sort of proof works for IZF.

Assuming that each function from N to N is computable has a number of consequences that are liable to be unfamiliar. Since it's contradictory with the law of excluded middle, you have to be ready to work with intuitionist logic.

Here, though, the relevant consequence is that the function mapping those Turing machines that compute real numbers to the real numbers they compute would be a SURJECTION from a SUBSET of the natural numbers to the real numbers. Please, now that we've gone through this more than once, either don't try to quote the result, or remember these key details. It doesn't follow that there's a bijection. I don't happen to know whether it's consistent to assume there exists a bijection between R and a subset of N. I suspect not, but I didn't succeed in working it out.

But most especially, there can't be a surjection from the whole of the naturals to the reals.

Keith Ramsay