Re: Uncountable sets in CZF?

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From: Ross A. Finlayson (*raf_at_tiki−lounge.com*)
Date: 08/30/04

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kramsay@aol.com (KRamsay) wrote in message
news:<20040830045814.04815.00000438@mb−m17.aol.com>...
> *raf@tiki−lounge.com (Ross A. Finlayson) writes:*
> *|Now we've seen in other threads that in IZF it is not inconsistent for*
> *|there to be bijections between N and R. Is that not so?*
> *>*
> *No, Cantor's theorem that for any function from N to R there exists*
> *an element of R not in the image is a theorem of IZF. Please just*
> *think of this as a mathematical fact, won't you?*
> *>*
> *I've seen proofs that it's consistent with various constructive formal*
> *systems to assume that each function from N to N is computable.*
> *If I'm not mistaken the same sort of proof works for IZF.*
> *>*
> *Assuming that each function from N to N is computable has a*
> *number of consequences that are liable to be unfamiliar. Since it's*
> *contradictory with the law of excluded middle, you have to be ready*
> *to work with intuitionist logic.*
> *>*
> *Here, though, the relevant consequence is that the function mapping*
> *those Turing machines that compute real numbers to the real numbers*
> *they compute would be a SURJECTION from a SUBSET of the natural*
> *numbers to the real numbers. Please, now that we've gone through this*
> *more than once, either don't try to quote the result, or remember these*
> *key details. It doesn't follow that there's a bijection. I don't happen to*
> *know whether it's consistent to assume there exists a bijection between*
> *R and a subset of N. I suspect not, but I didn't succeed in working it*
> *out.*
> *>*
> *But most especially, there can't be a surjection from the whole of the*
> *naturals to the reals.*
> *>*
> *Keith Ramsay*

Cantor−Schroeder−Bernstein: it works both ways.

What that means is that one of the reasons that people call the reals
uncountable is because they've figured out a bijection between the

reals and the powerset of the naturals, thus they reason that there are no bijections between the reals and the naturals, because Cantor−Schroeder−Bernstein says the existence of a surjection either way between two sets is proof of the existence of a bijection between those two sets.

That is to say, the existence of a surjection from A to B and from B to A implies that A and B are equivalent, and as well from A to B to C and C to B to A through composition.

That implies it is not a mathematical fact and to promote the other view as gospel, immutable, written in stone, etcetera, would thus be deceitful. I'm rather angered that you would suggest the acceptance of a mathematical falsehood as mathematical fact. Wouldn't you be?

Reexamine the claim about there being only one or none proper classes. Consider why that demands dual representation of the ur−element as both zero and Ord, regardless of whether Ord is N. (Steve, infinite sets are equivalent.)

Unitize the analog! It's better to have a tool, even a primitive tool, to measure sparse points of the continuum, than none, and bar further consideration of the matter by fiat.

Why don't you consider that the direct sum of infinitely many copies of N is zero?

>From Dave Seaman's giveaway that an "uncountable" set is a countable union of countable sets, to that the constructivist powerset mapping argument is shoddy or fails, to these metatheoretical hand−wavings of yes and no and ignorance fo the excluded middle, we've seen some of what I would call progress in the state of discussion of these issues here on sci.math, an open public forum where the participants are often professional mathematicians and as such loathe to promote untruth, particularly in the stark, concrete world of mathematics, that is already a paradise with no need for the transfinite: inconsistent and thus hellish.

So, in a post−Cantorian world, where it is accepted that Cantor's work is among strong developments of foundations a hundred years ago, Cantor is acknowledged as a discoverer and mathematician, and there's only one way to biject the reals and naturals, and a horsedrawn carriage leaves shit on the road, whereas an electric Fiat could go 300 miles per hour.

There are plainly rationalizations that you don't have to accept the powerset mapping result of Cantor as mathematical fact. To do so is more politically correct than honest for the urbane sophisticate uncaring of the truth, or card−carrying party member, except that it is as well pretentious and vapid, and the seasons change.

You're intelligent and understand the basic tenets of rational discourse, and by now you're probably familiar with the rather few arguments of transfinite set theory: assume to argue that infinite sets are equivalent.

You may well be able to have it both ways, in a post−Cantorian world. You have a choice.

The numbers are uncaring. Only software cares.

Have a nice day. Regards,

Ross A. Finlayson