Re: Uncountable sets in CZF?

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raf@tiki−lounge.com (Ross A. Finlayson) writes:

<snip>

>By "plain set theory" I meant theory where the only primary objects
>were sets as opposed to a theory where numbers are primary objects.

ZF and ZFC are therefore what you call "plain set theories". In ZF and ZFC, numbers are defined AS SETS. I wonder where you got the erroneous idea that numbers were primary objects in ZF and ZFC???

A natural number is defined to be either the empty set or a successor ordinal whose elements are all either empty or successor ordinals, a set n is a natural number if

n is empty or [n is a successor ordinal and for all m in n (m is empty or m is a successor ordinal)].

Equivalently, a natural number is an element of the smallest limit ordinal. The smallest limit ordinal omega is a model of Peano's Axioms with 0 being given by the empty set and the successor function being given by S(x) = x u {x}, where for sets A and B, A u B denotes the union of A and B. Addition on N (the set of natural numbers, i.e. omega) is defined by recursion by

m + 0 = m,

m + S(n) = S(m + n).

Multiplication on N is defined by recursion by

m.0 = 0,

m.S(n) = m.n + m.

Define the relation == on N x N by (a,b) == (c,d) iff a + d = b + c, then == is an equivalence relation on N x N. Define Z = (N x N)/==, and define an integer to be an element of Z, i.e. an integer is an equivalence class in N x N, and therefore a subset of N x N. Addition is defined on Z by

[(a,b)] + [(c,d)] = [(a + c,b + d)].

Multiplication is defined on Z by

[(a,b)].[(c,d)] = [(a.c + b.d,a.d + b.c)].

Both addition and multiplication on Z are well−defined. N is embedded in Z by mapping m to [(m,0)].

Define the relation == on Z x (Z−{0}), where 0 in Z is identified with 0 in N by the embedding, by (a,b) == (c,d) iff a.d = b.c, then == is an equivalence relation on Z x (Z−{0}). Define Q = (Z x (Z−{0}))/==, and define a rational number to be an element of Q, i.e. a rational number is an equivalence class in Z x (Z−{0}), and therefore a subset of Z x (Z−{0}). Addition on Q is defined by

[(a,b)] + [(c,d)] = [(a.d + b.c,b.d)].

Multiplication on Q is defined by

[(a,b)