

Re: Uncountable sets in CZF?

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In article <cgkfng\$abo\$1@bunyip.cc.uq.edu.au>, D.McAnally@i'm_a_gnu.uq.net.au

(David McAnally) writes:

|The Platonic attitude that there is exactly one set \mathbb{R} of real numbers is
|not fully appropriate.

I disagree. Even while wearing my constructivist hat I still disagree.

If you're going to adopt the attitude that there isn't one, which is up to you, you still are left with the fact that as far as ZF is concerned, there exists a unique set \mathbb{R} of real numbers. So if you're in the middle of doing some reasoning inside ZF, your reasoning should respect that. If you are not in the middle of doing some reasoning based on ZF, then a few words about what you are doing instead might be in order.

I think trying to think of the reals as not being unique can easily lead to confusion if you don't carefully partition such beliefs off from the mathematics one is trying to do, if it's in a system that regards the reals as being unique. If there's some alternative metatheory in which the unique existence of \mathbb{R} is not a theorem, but the results from model theory you want to use are, it should be named.

|One cannot assume that there is exactly one set \mathbb{R}
|that must be common to all models of ZF,

Of course.

|or that models of ZF are flawed
|on the basis that \mathbb{R} can be altered by taking a generic extension.

It depends on what you mean by "flawed".

Certainly it's possible for a model of ZF to have the real line as its \mathbb{R} . (If you like, think of it as a theorem of ZF. ZF doesn't have a theorem saying there are standard models of ZF, but adding an assumption such as the existence of a measurable cardinal, then there

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exists a model M of ZF where the set of reals relative to M is \mathbb{R} .)
Such a model might be considered more perfect.

In article <cgfu52\$uev\$1@bunyip.cc.uq.edu.au>, D.McAnally@i'm_a_gnu.uq.net.au
(David McAnally) writes:

[For myself, the easiest way to understand generic extensions is through
[Boolean-valued models.

Nice of you to go to the trouble of explaining that in your posting.
It looks like you tersely covered a good fraction of the key points of
the development of boolean-valued model as it's done in textbooks.

A Boolean-valued model M^G can be arranged to behave the way you
describe; we can wind up with an element f in M^G such that
 $|f:Z^\wedge \rightarrow R^\wedge \text{ is a bijection}| = 1$, where Z^\wedge and R^\wedge are the counterparts
of Z and R in M^G , although R^\wedge is not what M^G considers to be \mathbb{R} . This
 f is essentially a function from $Z \times R$ to G . The most obvious choice
of f in this particular case is symmetrical under permutations of Z
and permutations of R (which both act on G).

The model can get away with thinking of q