semifinite measures

Source: http://sci.tech−archive.net/Archive/sci.math/2004−09/1617.html

---

From: Artur (*artur_at_opendf.com.br*)
Date: 09/08/04

```
Date: 8 Sep 2004 08:28:49 −0700
```

Hi all,

I'm trying to prove that, if u is a semifinite measure on a
sigma−algebra and E is a measurable set such that u(E) = inf, then,
for every every c>0, there's a subset F of E with c< u(F) < inf.
First, I noticed that, if u(F) = c < inf for some subset F of E, then
the additivity of the measure implies that u(E/F) = u(E) − u(F) = inf
− c = inf. Since u is semifinite, it follows E/F contains a subset G
with 0<u(G)<inf. And since F and G are disjoint subsets of E, H = F U
G is a subset of E such that u(H) = u(F) + u(G) > u(F) < inf. This
shows for every measurable subset F of E with finite measure there's a
measurable subset H such that u(F) < u(H) < inf. But this doesn't show
the desired proposition.
If, by way of contradiction, we assume there's a c>0 such that u(F) <=
c for every subset F of E with finite measure, then there exists s =
supremum S = {u(F) | F is a subset of E with finite measure}. In
virtue of the first conclusion, this implies s is not in S. But this
doesn't lead to a contradiction, we don't conclude S = inf.
Could anyone give a hint, please? Maybe we could construct an
ascending sequence of sets F_1 < F_2....< F_n+1... whose union is E
and such that u(F_n+1) − u(F_n) >= k for some k>0.
Artur