Re: Raatikainen's critique of Chaitin

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In article <chn0t8$667$1@charm.magnus.acs.ohio−state.edu>, Nath Rao
<RnNaDthOrMao@yahoo.com> writes:
|BTW, Changeaux, a biologist IIRC, is (I am sure) a realist with regard
|to the phenomenal world. I suspect he didn't know enough to ask where
|such things as "the" category of (all) C^*−algebras and K−theory
|functors live. I just have a problem coming with an answer to such
|questions that can be squared with the way people actually think and
|talk. [Yes, Grothendieck universes are a way around if you don't mind
|excessively strengthening the consensus axiom system, but as A. Levy
|pointed decades ago, people talk as if they are referring to "the"
|universe, rather than >a< partial universe. The other way out is alien
|to most practicing topologists/algebraic geometers etc, and will fail
|anyway the moment somebody 'discovers' unavoidably impredicative
|functors 'in nature'.]

Well, it seems to me that people naturally think of there being such things as properties of sets, and treat proper classes as being the extensions of those properties. I think they feel okay about this as long as the argument can in principle be rephrased so as not to depend on treating those properties as objects in their own right. Many arguments can of course be rephrased by substituting specific predicates where references to proper classes are made. I'm not sure at what point such a workaround ceases to work.

By "impredicative" I suppose you mean a functor which, regarded as a proper class, can't be defined by quantifying merely over sets, i.e. isn't expressible in the first−order language of set theory (of the cumulative hierarchy).

I find it a little hard to imagine how this would arise in "nature". I can give an example of such a property; being (the Goedel number of a first−order sentence that is) true (in the cumulative hierarchy) cannot, by Tarski's undefinability of truth, be defined. I don't think people would have too much trouble with accepting a definition that was in those terms.

One can prove a lot of theories of proper classes (and metaclasses of proper classes and so on) to be consistent relative to these same large cardinal hypotheses. Just pretend that the extension of ZFC is talking about the sets below some large cardinal, and that the proper classes and so on belong to the next few ranks. This shows that the extension of ZFC talking about all those extravagantly big things (way too big to be sets, even) has no more consistency strength than axioms "merely" asserting the existence of some not−so−endless infinite cardinals. I think this is in some sense the way people use the workarounds using Grothendieck universes: show that we're not running into a trap by the manner in which we're talking about "large" categories. Then revert to using the same lingo, but thinking of it as referring to properties of all sets, or to proper classes.

I still remember the day someone defined a K−group to me, starting by saying, "take the free abelian group with a generator for each module over the ring R...".

Keith Ramsay