

Re: A simple question about integers

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From: Angus Rodgers (angus_prune_at_bigfoot.com)

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On Wed, 15 Sep 2004 08:31:08 +0200, Gottfried Helms
<helms@uni-kassel.de> wrote:

>Am 15.09.04 04:01 schrieb Eray Ozkural exa:
>> Angus Rodgers <angus_prune@bigfoot.com> wrote in message
>> news:<5lvdK0lnnki8hpsrlcj2ajfjftom9uud2@4ax.com>...
>>
>>>The long answer is also `no'. What I mean is that even
>>>if you try to embed \mathbb{Z} into a larger structure which has
>>>some of the same properties as \mathbb{Z} , but which also contains
>>>`numbers' with infinite decimal expansions, such as:
>>>
>>> ...57803170965
>>>
>>>– leaving aside the question of what this would `mean' –
>>>
>>>then you are not going to be able to define addition in
>>>your new structure in any way resembling addition in \mathbb{Z} :
>>>because in a case where there is no `carrying' you will
>>>certainly want to perform addition digitwise (e.g. in \mathbb{Z} ,
>>> $320 + 539 = 859$ because $0 + 9 = 9$, $2 + 3 = 5$, $3 + 5 = 8$),
>>>but this forces you to define the sum of the above number
>>>with the following (equally reasonable-looking) `number':
>>>
>>> ...42196829034
>>>
>>>as:
>>>
>>> ...9999999999
>>>
>>>and you will not even be able to add 1 to that `number'
>>>in any sensible-looking way.
>>>
>>>I don't see any way around this. Do you?
>>
>>
>> I don't either.
>>

- > *There are two more problems with that:*
- > *such a definition, written down or not, introduces the*
- > *concept of divergent series, where we know, that results*
- > *may be arbitrary depending simply on the order of operations.*
- > [...]

That's not necessarily a problem, because there is no problem with the concept of a formal power series – considered simply as the sequence of “detached coefficients” of the powers of a notional “indeterminate” (say θ).

(This is a standard construction, identical with the standard construction of polynomials, except that the requirement for all but finitely many of the coefficients to be 0 is dropped.)

One could try to define, quite formally,

$$\left(\sum a_n \theta^n\right) + \left(\sum b_n \theta^n\right) = \sum (a_n + b_n) \theta^n$$

$$\left(\sum a_n \theta^n\right) \left(\sum b_n \theta^n\right) = \sum \left(\sum a_{i} b_{n-i}\right) \theta^n$$

where a_n, b_n are arbitrary integers, and there is an equivalence relation by which one can reduce any formal sum to one in which the coefficients are single digits relative to the chosen base of numeration r (e.g. 10), perhaps by taking the quotient by the principal ideal $\langle \theta - r \rangle$.

(This may be nonsense, of course – I stopped trying to follow it through, when I realised you couldn't always carry 1 when all but finitely many digits are $= r - 1$, and you couldn't restrict yourself to sequences without this property, as is shown by the example above.)

- > *the more serious problem in that way is, that with arbitray*
- > *sequence of digits we even cannot define a greater/smaller*
- > *order between two such numbers... since they have no*
- > *highest digit.*

I don't think this is obvious. One could at least *try* to define a structure of “infinite nonnegative integers”, considered as infinite decimal expansions (or expansions to another base); define $a < b$ if and only if there exists c such that $a + c = b$; and then try to prove things about this relation (such as totality, i.e. either $a < b$, $a = b$, or $a > b$) – with the hope of then being able to embed this “big N ” structure into a “big Z ” structure, in much the same way as Z can be constructed from N .

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But none of this is any good if you have a ``number", n , for which you cannot even define $n + 1$ – so it all looks pretty abortive, to me at least.

(Pity – it almost looked like fun for a while.)

--
Angus Rodgers
(angus_prune@ eats spam; reply to angusrod@)
Contains mild peril