# Re: Countably infinite Hausdorff topology?

***Source:*** http://sci.tech−archive.net/Archive/sci.math/2004−09/4034.html

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***From:*** David C. Ullrich (*ullrich_at_math.okstate.edu*)
***Date:*** 09/17/04

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Date: Fri, 17 Sep 2004 08:44:07 −0500
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On Fri, 17 Sep 2004 12:04:04 GMT, "shedar" <no_one@nonesuch.com>
wrote:

*>"David C. Ullrich" <ullrich@math.okstate.edu> wrote in message*
*>news:ir9gk0p7vvkua3budg8ems9n17ch4tqtg6@4ax.com...*
*>> On Tue, 14 Sep 2004 14:04:39 GMT, "shedar" <nobody@nonesuch.com>*
*>> wrote:*
*>>*
*>> >"Robert Israel" <israel@math.ubc.ca> wrote in message*
*>> >news:ci50j1$hsf$1@nntp.itservices.ubc.ca...*
*>> >> In article <fkl1d.6375$G03.1882402@news4.srv.hcvlny.cv.net>,*
*>> >> Stephen J. Herschkorn <herschko@rutcor.rutgers.edu> wrote:*
*>> >> >A standard exercise is to show that any infinite sigma−field has*
*>> >> >cardinality at least 2^omega. That got me thinking about topologies.*
*>> >>*
*>> >> >Does there exist a Hausdorff space whose topology is countably*
*>infinite?*
*>> >>*
*>> >> Suppose X is a Hausdorff space with an infinite topology. In*
*>particular,*
*>> >> X is infinite. Then, with at most one exception, each point of X has*
*>> >> a neighbourhood whose complement is infinite (i.e. if there were*
*>> >> two points x and y whose neighbourhoods all had finite complements,*
*>they*
*>> >> could not have disjoint neighbourhoods, violating the Hausdorff*
*>> >> requirement). So let x_1 be a point of X with an open neighbourhood*
*>U_1*
*>> >> whose complement is infinite. Now X \ U_1 is also an infinite*
*>Hausdorff*
*>> >> space, and the same reasoning applies to it: there is x_2 in X \ U_1*
*>> >> with an open neighbourhood U_2 such that X \ U_1 \ U_2 is infinite.*
*>> >> Moreover, again using the Hausdorff property we m*