

# Re: Skolem's Paradox and why is math the way it is?

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- > */And for the mathematicians that think this is unnecessary, then how*
- > */do we know everyone is doing the same math if the axioms don't*
- > */describe "real numbers" uniquely?*
- >
- > *This is not a job the axioms were ever meant to do.*

Then let's use IF logic instead, since you seem to be aware of it. I'll define a class of objects to be "intuitively infinite" if the objects of the class make the following statement true (in the game-theoretical symantics sense).

there exists z such that, (For all x,y):  
(((x=y)or(not(x=y)))and(not((x=y)and(not(x=y)))))) and  
((there exists U ind. of y) such that, ((there exists V ind. of x)  
such that, (((x=y)or(not(U=V))) and (not(U=z)))) )

I'm VERY comfortable with this definition, because I require that the other person's interpretation require a winning strategy, no more, no less. I'm pretty sure than any model of set theory is intuitively infinite, as are the elements of the natural numbers IN any model of set theory. And since I did NOT use set theory to describe the definition of intuitively infinite, then hopefully we all agree where a class is intuitively infinite.

This is much better to me, than the concept of "countability" which is only true IN A MODEL based on whether a set called a bijection (IN A MODEL) exists with certain properties. "Countability" then is a property of a set that depends, not on the quantity of elements in the set, but instead on whether the MODEL in which the set lives contains some other set. Countability is an extrinsic, contextual definition.

- > *You're faced with a sort of bootstrapping problem. Some mathematical terms*
- > *are of course defined in terms of more elementary ones. But somehow you*
- > *need to get the whole system started up. How can we explain it without*
- > *getting into an infinite regression?*

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I'd prefer to start with IF-logic, because the concept of the winning strategy requires us to share the parts of an interpretation that we care about for defining things.

- > *It's actually a problem with language generally, not just mathematics.*
- > *People just tend to notice it in a mathematical context. The way we*
- > *bootstrap language is a somewhat messy process in which we don't at all*
- > *"define" the initial vocabulary we work with. Instead, we have a process*
- > *of informally becoming acquainted with words and their meanings*
- > *by observing how they are used.*

I have the SAME problems with bootstrapping in "everyday" language, people say ambiguous stuff ALL the time and expect you to know what they are saying. And it's not like I'm just dumb, I've watched other people, and sometimes they just interpret something vague in a way different than what the speaker intended. In physics we have to go all the way back to an operational definition, which is based on the requirements you would have to meet in order to "repeat" an existing experiment. Basically you DO repeat a subclass of experiments yourself until you demonstrate that you interpret the PAST experiments the right way. If LATER it is revealed that your interpretation and someone else's were different and both worked for the old experiments, THEN that's GREAT because you can devise an experiment that distinguishes between the two and DO it and THEN you've PROVED something about the real world. So in physics this is an asset. In mathematics there is no real world to appeal to, so I have trouble figuring out how I tell what is real in mathematics. Did that make any sense to you?

- > *The concept of a model of a set of axioms isn't a good starting point for*
- > *bootstrapping your way up, because it depends upon our already having a*
- > *concept of a set of elements (for the domain of the model). If you don't*
- > *have such a notion, then it doesn't make any sense to talk about models*
- > *of axiom systems. On the other hand, if you do have such a notion, then*
- > *not all of the mathematics you are doing is created within one of these*
- > *models you are considering; the mathematics you used to discuss the nature*
- > *of these models is defined independently of them.*

I had figured that the models of ZF had a domain in ZF, because mathematicians don't seem to like to use anything different than ZF, this is VERY bad for this discussion, because the other posters keep jumping around and using a word defined in one ZF (the base model) in another ZF (the model).

I'll try to describe what I think is going wrong, and you can either tell me where and when I make a mistake, or if it's all correct you can tell me how I'm interpreting it all wrong. The point is that if we first PRETEND that ZF is naively real there is a set (in the naive ZF) called the naturals, and this set CAN be interpreted in such a way as to have someone write the word "empty set" on the number 0 and "naturals" on the number 1 and the word "reals" on the number 2 and

the name "set containing 2 and 5" on the number 3 and so on. Eventually the names will just be lists of how the set are asserted to exist from the axioms of ZF, but after all the name writing, someone could hide the other members of the naive ZF and show someone else the naturals with the underlying numbers covered up (so that they couldn't tell it was a model and just saw the words written on the elements) and then sat there and everytime someone asked for a set because the axioms implied it existed, the guy who painted the words could hand him the right number with the right words on it. That sounds like the countable model to me. People object because they think I'm saying that this property of "which natural is behind every set we work with" should somehow BE encoded in some set in ZF, when in reality it can't be encoded in such a such. That is bad for the naive people, because it shatters their platonic belief that THEY are dealing with a REAL universe of ALL sets that are possible, instead of just a small class of all sets possible.

> *To avoid an infinite regression, you need to have some base theory that  
> can be understood in its own terms, and not translated into a language  
> of models of the theory. Perhaps you don't believe the axioms of ZFC,  
> but believe they are consistent. Then you can prove theorems in ZFC, with  
> the understanding that you don't believe these results, but only believe  
> they hold inside models of ZFC. But the theory in which you reason about  
> the models of ZFC (your "metatheory") has to make sense or else you have  
> just procrastinated the problem of interpretation for one step.*

We can avoid bootstrapping AND IF-logic, and I think just three iterations should make my point clear. So imagine you use ZF as a base system, and you pick a countable model of ZF INSIDE of it on the domain `Naturals_in_ZF` and call the model "naive-ZF", and then inside NAIVE-ZF you pick a countable model of ZF on the domain `Naturals_in_naive_ZF` and call the model "the stupid J.E. model, that is only interpreted by stupid people like J.E. because they are so boneheaded" or `tsJEmtiotbsplJEbtasb` for short. Everyone except me seems to say that `tsJEmtiotbsplJEbtasb` is a "contrived" model of the ZF axioms and that the element of the `Naturals_in_naive_ZF` that represents the reals in the `tsJEmtiotbsplJEbtasb` model is WRONG because the element of naive-ZF that (in naive-ZF) represents the bijection (in naive-ZF) from the set of `Naturals_in_naive_ZF` (not from the element of `Naturals_in_naive_ZF` that represents the naturals (in `tsJEmtiotbsplJEbtasb`)) to the subset of the `Naturals_in_naive_ZF` that represents the class of singleton subsets (in `tsJEmtiotbsplJEbtasb`) of the reals (in `tsJEmtiotbsplJEbtasb`), that is not ITSELF a member (in `tsJEmtiotbsplJEbtasb`) of `Naturals_in_naive_ZF` ('cause it's a bijection and not a natural number). So everyone except me, APPARANTLY thinks that because naive-ZF has an element that represents an injection from the naturals TO a proper subset of the naturals that this means that the class of singleton subsets of the reals is either countable or not countable. That seems silly. Such a mapping proves that the universe of sets in my model is INTUITIVELY INFINITE, and NOTHING more.

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To me, a set is countable or not depending on the bijections between sets IN THE MODEL and a class of sets in a model is countable or not depending on the bijections that include the domain of the model in the BASE MODEL. These are completely different things. But the point is that in a MODEL we can NOT tell what bijections exist in the BASE model.

If we use ZF as a base system, then the idea of isomorphisms between models (in ZF) is going to be an ARTIFACT of the base system (ZF), not always related to the properties of the models themselves in whatever intrinsic properties they have. The cardinal system IN the countable model is clearly an artifact of missing bijections IN the model, but there is no way to "tell" that the "base system" isn't itself a model in a bigger person's base system, so OUR class of isomorphisms of models could look really stupid to that person.

- > */How do we know which model is the "intended" interpretation of set*
- > */theory?*
- >
- > *A complete answer would require a long discussion of set theory. But take*
- > *a simpler special case.*
- >
- > *How is it possible for a student to know that when the instructor says*
- > *"let  $f$  be a function from  $Z$  to  $Z$ ", the intended meaning is what the*
- > *student thinks it means? Really, it's essentially the same way as we*
- > *come to know that when someone says "apple" they probably mean what we*
- > *think they mean.*

Each student picks up their favorite model of the axioms the instructor and students agreed to share. Each model has an interpretation of these axioms, so each student in their individual model figures out which axioms have to be invoked to assert the existence of the set the instructor talked about, and selects the element of their model that invoking those axioms tells them in their model to select. Then each student calls that  $f$ , and looks back at the instructor to see what happens next. Every well-formed statement that must be either true or false in every model can be evaluated in every model. If everyone agrees that something is true in their model, then it is said to be "true" of the axioms. If it is false in all models, then it is said to be "false" of the axioms. If it is true in some models and false in other models, then it is said to be "independent" of the axioms.

This is what has ALWAYS happened in EVERY class I've ever taken. The axioms are ABOUT things that were not EVER defined EXCEPT in how they related to the axioms. If someone talks about "points" and "lines" in geometry you can imagine whatever you like as you make your model, but it had better follow the axioms of geometry that you are sharing with other people if you want to talk about what is true or false or independent of those axioms you share.

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There is NO way to tell who is using WHAT model of ANY axioms, because we can't get inside their head, but it doesn't matter, because the TRUTH of a axiom system is about what is SHARED in ALL models.

- > *It's always possible to come up with unintended interpretations that are*
- > *formally consistent. If every time I say "apple", you interpret it to mean*
- > *"apple if in the northern hemisphere, orange if not", your interpretation*
- > *is probably going to stay consistent with what I am saying for a long time.*
- > *But it's a far less natural interpretation, which is how we are able to*
- > *know that it's not what I mean.*
- >
- > *I can try to block such misinterpretations by providing more explanations*
- > *of my terms, but ultimately there's no way for a speaker to prevent a*
- > *listener from cooking up an artificial interpretation different from the*
- > *one intended. Nonstandard models are just another way to cook up such*
- > *interpretations. For any given countable model of ZF I can name, the*
- > *interpretation of mathematical statements as being about that model is a*
- > *much more artificial interpretation than thinking that what I mean by*
- > *"apple" in the southern hemisphere is "orange".*

If you compare two models, the only artificial appearances come from comparing them as two translations of a common "base" model. For instance, if  $R^{(4,1)}$  is interpreted as a hyperbolic non-Euclidean geometric space with the axioms of hyperbolic geometry and INSIDE it you made TWO models, model #1 takes the points, lines, and planes of  $R^{(4,1)}$  that are contained on a fixed 3D hyperbolic 3-plane, then one can construct model of 3d non-Euclidean geometry, on the other hand if you took ALL the 3D hyperbolic 3-planes in  $R^{(4,1)}$  and interpreted EACH as a point, then one could construct a model of 3D EUCLIDEAN geometry. The non-Euclidean model LOOKS easier, but it is no better than the Euclidean one.

- > *Moreover, the interpretation of statements as being about models relies*
- > *upon there being a more fundamental level of mathematics in which it's*
- > *possible to talk about the models in the first place.*

But labeling some models COUNTABLE and others NOT, assumes that the concept of countability (or cardinality) in the BASE model is meaningful. There is no basis for that, and it seems pretty silly to me at least, when making a model of oneself because you are assuming the validity of the concept you are then talking about. I can see that the models of ZF are "intuitively infinite" with the definition in IF logic since that is outside ZF completely. But I don't see any bigger kind of infinity. It seems like cardinal theory goes like this: ... you go along counting, and then when you "forget" a number (don't have a bijection IN your model), you jump to some infinity that you say is different.

- > *Why do we care about "uncountably many" real numbers when in reality*
- > *there are only a countable number that we can "prove theorems about"?*
- >

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- > *You've been writing as if you knew of a well-defined countable set of*
- > *real numbers that includes all the ones we can prove theorems about. I*
- > *don't think there is such a thing.*

I'm writing about the set of real numbers IN the countable model. Since it IS a model, anything true of ZF is true IN the model, with the interpretation IN the model. I didn't say it was "well-defined and countable" IN the model, but it IS outside the model (in the base model), which means we NEED NOT appeal to ANYTHING more than a countable (in the base model) set AND an interpretation.

- > *Consider for example the set of reals you considered in another posting,*
- > *the ones proven to exist in ZFC or maybe it was ZF. This is a countable*
- > *set, true. But it's not a set that can be defined in ZFC. The property*
- > *of being definable \*in those terms\* cannot itself be defined in those*
- > *terms. There's a subtle catch.*

I agree that it is not a set IN the model. I think that is a problem because it is a logical description OF a bijection that does NOT exist in the naive interpretation of "all bijections". Am I really not being clear here?

- > *We can extend the language of ZFC so as to make it possible to define that*
- > *countable set of reals, but when we do, we are able to define more reals*
- > *in that extended language than we could in ZFC itself.*

Once we do that, we'd have to close ZFC (make ZFC\*), and AFTER that we could make a countable model of THAT closure, and then we could make a bigger set theory ZFC\*\*, but we are only adding countably many sets each time, and this process is a PHYSICAL process with two people making theories tit-for-tat, and transfinite induction doesn't apply, so they can do that until the cows come home and EACH theory will have a countable model. Just like if we added the Godel theorem as an axiom, then there would be another truth that could be consistently added, and after we added that, then another one.

- > *It's not clear to me that there's such a thing as absolute undefinability.*
- > *Goedel had a few remarks on the issue. He observes that if we could define*
- > *such a thing as absolute undefinability, there aren't any ordinals that are*
- > *absolutely undefinable. (If there were any, there would be a least one.*
- > *But being the least ordinal satisfying the definition would define it.)*
- > *So anything that can be defined in terms of ordinals similarly can't*
- > *be categorized as absolutely undefinable. That's a lot!*

Different things will be definable in DIFFERENT models, it's not a property of the axioms. Hence it is not TRUE of the axioms that such-and-such is the smallest undefined thing, because DIFFERENT things can be defined in many models. Naive set theory is descriptively INCOMPLETE because different models can DESCRIBE and DEFINE separate DIFFERENT things that can't be described in other models. That makes me LEERY of naive set theory (standard

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interpretations), but before jumping on some prejudice of mine I'd like to know WHAT the differences are in these models. However when I talk to people, they won't discuss this because they have their OWN prejudice about what model is best and think that \*I\* am prejudiced to harbor ANY doubts.

- > /Warning-- if the following question offends you, just pretend that it
- > /is rhetorical and merely answer the OTHER questions. --Is this some
- > /kind of Platonic conspiracy or vestigial holdover?
- >
- > /It's not a conspiracy, because those who regard Platonism in mathematics
- > /as valid are simply supporting what they see as truthful. On the other
- > /hand, those who see it as a vestige are merely hoping that the truth,
- > /as they see it, will win out.

Why talk about things that people CAN argue about. That's not the point of mathematics, that's the point of religion. If someone talks about something vaguely enough that people can disagree about whether it's a snake or an elephant, then why not talk more clearly. Why not have descriptively complete theories?

- > /> Second-order logic (the recursively axiomatizable version of it,
- > /> anyway) is still deductively incomplete, so it doesn't make much
- > /> difference.
- > /
- > /Have you studied IF-friendly logic, and why do we use set theory
- > /instead of just pure logic?
- >
- > /As far as I can tell, "IF-friendly" logic is a topic considered only
- > /by Hintikka. By the way that it handles dependencies between variables,
- > /it permits one in effect to assert the existence of functions having
- > /certain properties. As far as most of the important differences between
- > /first-order logic and second-order logic are concerned, that puts it
- > /closer to second-order logic.

I think it might break the Skolem paradox without opening up a second order version of it. If so, that makes it very good. It also has a logical principle that beats the Axiom of Choice hands down and it requires ALL interpretations have the properties \*I\* want interpretations to have. I've never met other people (besides myself) who like it though. When people write negative reviews (yes I know that Hintikka made some easily correctable errors) they send them to me too, because I say that I like it. But the arguments against it don't seem very useful to a physicist. I want something big enough to do physics and small enough to not have parts that are unrelated to physics, and I want different physical models to be different only based on testable differences. That's all I want.

- > /The reasons why set theory is used are more practical than theoretical.
- > /If you want to reexamine the 19th century development of real analysis,
- > /you can feel free to consider whether they could have accomplished the

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- > *same things a different way. If they had known of a better way to do it,*
- > *they would have taken it.*

They can't do the EXACT "same thing" DIFFERENTLY. And since my standards are usefulness for physics, if I tried to apply those standard all at once, then I'd have to create a TOE from scratch out of logic with my bare hands, that's a TALL order. Surely there is some partial work that can be done, on making a solid foundation?

- > *One could use second-order logic as a starting point, and it wouldn't*
- > *necessarily make much difference. We could write down second-order axioms*
- > *that uniquely specify a "large chunk" of the universe that the language*
- > *of ZF is meant to talk about. By a "large chunk", I mean that it includes*
- > *the set theoretical representations of all the objects that mathematicians*
- > *other than set theorists consider: all the reals, all the functions from*
- > *the reals to the reals, and so on.*

I don't like classifying parts of classes based on WHO considers it, that's not scientific. Science needs to be "in theory" repeatable BY anyone at ANY time and in ANY place. Set theorists already are the "only ones" (I guess you and I are set theorists now) who talk about the class of ordinals and the operations of ordinal arithmetic and such.

- > *Is there any reason that we want sets*
- > *with more numbers than we can prove things about. It seems silly to*
- > *use a bigger, more complicated model than necessary. As a physicist I*
- > *don't want more numbers in my model than required.*
- >
- > *Bigger and more complicated do not go hand-in-hand. The set of reals*
- > *provably existing in ZFC is a relatively complicated set. The step in the*
- > *construction of the real number system that causes the big "expansion"*
- > *in "cardinality" is that of considering an arbitrary function from the*
- > *natural numbers to the rational numbers. The notion of an arbitrary*
- > *function from one set to another is much simpler than almost any notion*
- > *of a function that can be defined by restricted means.*

But what you REALLY add to ZFC to make it "bigger" is not the CLASS of "an arbitrary function from the natural numbers to the rational numbers." but the class of "functions from the natural numbers to the rational numbers, consistent with this particular model of ZFC", each model THINKS of ITSELF that there are "ALL" because it added enough to APPEAR closed, but more COULD exist in a DIFFERENT model, just like in NSA.

- > [...]
- > *No, we do NOT because every theorem can be interpreted in the*
- > *countable model, where there are NOT uncountably many singletons (in*
- > *the class of all sets). Why isn't the countable model the "intended"*
- > *model, and why can't we fix the axioms so that this illusion of more*
- > *members goes away?*

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- >
- > *That should be "a" countable model. There are uncountably many of them!*
- >
- > *The only way that it makes sense to call a model a "countable model" is*
- > *if you believe in the existence of a one-to-one correspondence between*
- > *the natural numbers and the elements of the model. That correspondence*
- > *is external to the model.*

A model is countable (IN THE BASE MODEL) if there is a bijection (IN THE BASE MODEL) between the naturals (IN THE BASE MODEL) and the DOMAIN of the model (IN THE BASE MODEL). But this set (IN THE BASE MODEL), implies (if it real) a LOGICAL correspondance (not a set in any particular model, but a LOGICAL correspondance) between the class of {strings of the form "0" followed by "+1" an arbitrary number of times} and the class of {reals IN THE MODEL}, which is non-intuitive with the notion of ALL correspondances already being IN the domain of the model. Am I really being this unclear?

- > *It's this bootstrapping issue again. If you don't believe in any*
- > *mathematical objects to begin with, you don't ever get to the point where*
- > *you believe that there are such objects as "models of ZFC", so there's*
- > *no question here. If you do believe in certain mathematical objects, then*
- > *by the time you get around to defining the notion of a countable model of*
- > *set theory, the model is then an object that exists in a context of other*
- > *objects, which belie its claim to including "all sets". From the model*
- > *itself, we can construct additional sets that aren't in the model.*

ZFC is DESCRIPTIVELY incomplete and THUS it has models which can CONSISTENTLY be added to, and this contradicts the notion that these models are COMPLETE in the sense that they contain "all" sets. You seem to agree with me, but you state that you are argueing, that's very confusing to me.

- > *Consider: if ZFC is consistent, then there exists a model of ZFC in which*
- > *the false axiom "ZFC is not consistent" holds. Inside of itself, there are*
- > *no models of ZFC. But we know that if such a model exists, then it's not*
- > *the class of all sets, because the model is itself one of the sets that's*
- > *not a member of the model.*

You are saying that some models of ZFC can have axioms ADDED (so now they are models of ZFC+ for lack of a better name) that make it so that one can NOT make consistent models of ZFC withIN ZFC+. So what? "if such a model [which one] exists, then it's [which one] not the class of all sets" you lost me here, can you repeat that? The pronouns lost me, see the [] inserts in my quote of your statement.

- > /> *Well, even if you don't assume Platonism, it's still a theorem that*
- > /> *there are uncountably many real numbers. So surely that's a good*
- > /> *reason to "care" about uncountably many real numbers?*
- > /
- > /*I disagree, there is a theorem, that says for any given model, and any*

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- > */injection in that model, there exists another number that is in the*
- > */reals that has no preimage.*
- >
- > *The theorem doesn't say anything about models. Don't revise the language*
- > *by inserting your own interpretation of it into it.*

Theorem from axioms asserts truths that are true in ALL models. That's what it MEANS to be true from axioms. The diagonal theorem says that all injections IN A MODEL to the reals IN A MODEL have elements that are excluded. NO theorem says that there cannot be such a bijection IN THE BASE MODEL. That's all I'm saying, and from you wrote earlier, I think that \*you\* agree with me. If not, please explain WHY.

- > *In order to avoid an infinite regression, there has to be on some level*
- > *a language which is understood in its own terms, and not reinterpreted as*
- > *being only about models in some metatheory.*

IF-logic does that, because it incorporates the INTERPRETATION of statements with their TRUTH VALUE. ZF is descriptively incomplete and CANNOT do that because new truths about set theory can always be added. I \*suspect\* that all completely first order theories are like that, and that all completely second order theories are like that and so on. I \*suspect\* that IF logic is NOT like that, but no one seems to know one way or another because no one except me seems to CARE and \*I\* unfortunately am not very smart.

- > */It only proves ONE more number,*
- >
- > *It's important to remember that that's all that's needed. There seems to*
- > *be a strong temptation to think that uncountability should mean something*
- > *more than what the definition says. To resist the temptation, just keep*
- > *reminding yourself, it means \*nothing\* other than the fact that there is*
- > *a one-to-one function from the integers into the set, but no one-to-one and*
- > *onto function (i.e. one-to-one correspondence) between the set and the*
- > *integers.*

It shows that ONE number was missing from the bijection, it doesn't show that the missing bijection that lives IN THE BASE MODEL is of a HIGHER size by any definition of the word, unless you think the original bijection was finite, then the one IN THE BASE MODEL \*would\* be HIGHER size by some definition.

- > *You can, if you like, iterate the construction. If we have a sequence*
- >  *$a_0, a_1, \dots$  of reals, then we can get another sequence  $b_0, b_1, b_2, \dots$  of reals*
- > *by repetition. Then, however, since the  $a$ 's and  $b$ 's taken together are*
- > *still countable, we can keep going and get another sequence.*
- >
- > *This process can be continued transfinitely, to get a set of reals that*
- > *corresponds to each countable ordinal. That's  $\aleph-1$  reals. (The axiom*
- > *of choice was used.)*

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INSIDE the model you can use transfinite axioms to do create "the set of all [sic] reals" IN THE MODEL, but you ONLY created ONE set from using that axiom. The COUNTABLE model ALSO constructs a set when you use that axiom. The axiom (like the others in ZFC) ACTUALLY says that if you have a SET (in this case a set of SETs) that satisfy such and such property, then you can create another set with blah blah blah properties. So it says that ONE member of the universe (class of all sets in the model) causes ONE other element to exist (also be in the class of all set in the model) in the universe. There is NO \*ZFC\* axiom that creates a SET from a CLASS. Each time you use an axiom, you put IN a finite number of sets to the axiom and get OUT a finite number of sets. So your CLASS of sets in your universe keeps growing and growing, but never gets so big that there isn't a COUNTABLE (in the base model) model.

Do you understand that the class of all sets in a model can be very small as long as it doesn't include a reference to how small it is? Isn't that what the Skolem Paradox is saying?

- > / and since
- > /the set of all reals in ONE model is actually countable (from the
- > /outside) then it seems that the "uncountably many reals" are ghosts in
- > /the wind. Seems like fancy talk to say that the set of bijections
- > /between the reals is incomplete because our axioms made it so. Why
- > /not fix the axioms?
- >
- > No, the lack of a bijection is a fact. All that reworking the axioms
- > could do is disguise it or express it in different terms.

YOU are disguising that the class of all sets could be QUITE small, but that it is merely very hard to describe what exactly CAN be consistently in it and what CAN'T and so we pretend it is very LARGE by choosing to NOT think about is in it. The REASON we do that is because if you STOP to think about how small it is then you'll want to make it bigger, and then after everyone switches to the new one, someone ELSE could stop and think about it, and make it bigger, and that THAT process is a pain. But why not just use IF-logic from the beginning, then we have room for everything we WANT to describe that is verifiable, but we DON'T have to have cardinal arithmetic and discuss the LACK of bijections (because we don't have an excluded middle in the BASE theory (IF-logic), which I believe is the real cause of this confusion IMHO)

- > /Well, for instance, people say there are uncountably many reals,
- and
- > /I've seen the cantor diagonal theorem, but that just says that set
- > /theory doesn't contain abijection from the reals to the naturals
- > /within itself.
- >
- > Don't confuse a theory with the domain it talks about.

sci.math: Re: Skolem's Paradox and why is math the way it is?

I'm saying that the CLASS of all sets (which COULD be a set in the BASE model) is NEITHER countable, NOR uncountable BECAUSE it is not a set and there are no surjections OR injections to NONSETS (in the model). So the CLASS of reals is not UNCOUNTABLE, it is NOT COUNTABLE which is DIFFERENT. People think that because the SET of reals is UNCOUNTABLE (in the model) that the CLASS of reals in the model is "uncountable" when it could REALLY be "countable" DEPENDING on the models relationship to the BASE MODEL where the cardinality of classes is determined.

- > *When we consider all bijections, it's like considering all orange*
- > *objects. It makes no more sense to ask whether we might be accidentally*
- > *considering only a subset of the bijections (when we say we're talking*
- > *about all of them) than it would to say, "But maybe when you talk about*
- > *'all orange objects' you are accidentally leaving some orange objects*
- > *out."*

You ARE leaving things OUT, because more functions from the naturals TO the reals COULD be added without breaking consistency. Specifically you only consider bijections that are ALSO sets. You aren't considered because you "called" your universe the class of "all sets", but that doesn't MAKE it so. If you look in the BASE MODEL, the idea of "being a set" is about whether it is in the domain of the model, which is arbitrary.

- > *| It actually only produces one other number, so no*
- > *|matter how many theorems you prove, and no matter how many times one*
- > *|uses the diagonal argument, one can only DEMONSTRATE countably many*
- > *|real (i.e. prove they exist) and in fact there is a countable set that*
- > *|one can interpret all the axioms of set theory on and IT contains a*
- > *|set of that can be interpreted as "the reals" and that set is*
- > *|countable. Why all the blather about uncountability then?*
- >
- > *Remember again the definition of uncountable.*

Countability has a SET theory definition and a MODEL theory definition. The SET of all reals is NOT countable (in the set theory sense), but the CLASS of all singleton subsets of the reals IS countable (in the model theory sense). The concept of uncountability is a bit like "being in my blind spot", it's perfectly real to you, but fairly meaningless to others not in the same EXACT location.

- > *|So if another physicist walked up to me and said that Occam's*
- > *|razor says the other numbers don't exist, how can I argue against him?*
- >
- > *I don't know exactly how Occam explained his razor, but it seems clear*
- > *that it's better to prefer \*simple\* theories than it is to prefer ones*
- > *that keep to a minimum the things that they have in them (if it's at*
- > *the expense of being less simple).*
- >
- > *All of these "trimmed down" sets of reals are more complicated than*

## sci.math: Re: Skolem's Paradox and why is math the way it is?

> *the full set of reals.*

I disagree about "complexity", all models are equally valid because the interpreting is the same. You PAINT the element to look like a set and INSTEAD of "looking inside the set" you THINK about what sets are in it. Since in reality sets don't float around our heads, we do the same thing in practise. When someone says that they have a countable model, what they mean is that they interpret each natural as a set, and TREAT it LIKE the set it represents, which is JUST as hard as interpreting each set as itself, BECAUSE you have to imagine it ANYWAY.

> *It's possible to write down axioms equivalent to the axioms of ZF that are reasonably short (in total length). So there's little hope of using Occam's razor to cut mathematics down to something simpler; such an argument would hinge on a fairly delicate weighing of theories that differ in complexity only a little bit. It's nothing like getting rid of an independent constant in a physical theory.*

Either math can be enlarged to introduce those bijection that currently aren't sets, or it can be cut down to eliminate But from a physicists standpoint, why use sets at all if we can make axioms about the actual elements of a physical model. THAT is what I want, and the ROUTE I'm exploring is to

> *I have real concerns (as a scientist) about how to make representations of mathematical objects, and I don't see how maintaining the fiction that real numbers that we can't describe somehow "exist" helps anything. It is platonic mathematicians NOT logicians that claim these numbers exist.*  
>  
> *The notion of "absolutely impossible to describe" has problems with it, as I suggested above.*

The naive "the class of singletons of the reals is uncountable even though it is a class and not a set" people are the ones that claim there are more reals than can be described. I claim that there are as many as there are natural numbers, but that they are MUCH harder to describe. I'm asking why I should hold Platonic views about their being more real numbers when it doesn't seem to either be true or matter in any way I know. That's why I wonder if it IS true and if it DOES matter, HENCE the title of this thread on sci.math

> *As for forgetting logica, after reading "The Principles of Mathematics Revisited" by Jaakko Hintikka, I'm leaning the other way and considering abandoning math and using logic instead, I don't know why math is supposed to be better.*  
>  
> *It contains mathematical logic.*

sci.math: Re: Skolem's Paradox and why is math the way it is?

I don't know what the definition of that is. What are you saying?  
That math contains mathematical logic and that is why it is better  
than IF–logic? That IF–logic contains mathematical logic and that is  
why it is worse than math? Something else?

- > / *It's not like anyone can prove that ZF is consistent.*
- >
- > *Perhaps you've heard of Goedel's second incompleteness theorem, which*
- > *precludes there being a proof in ZF of the consistency of ZF (unless*
- > *ZF is actually inconsistent). There's no real point in trying to work*
- > *out a loophole.*

I'm familiar that ZF cannot be proven to be consistent unless is  
actually isn't, AND it's true that I do have my personal doubts about  
whether it IS consistent. However I \*have\* learned that if I  
\*pretend\* it is consistent that it's WAY easier to get published.  
That doesn't make something true.

- > / *Has*
- > *anyone proven the countable model of ZF to be consistent? My first*
- > *and primary concern is understanding the physical universe and my care*
- > *for math or logic is as a foundation for that enterprise.*
- >
- > *Logicians don't differ all that much from any other mathematicians in*
- > *how they consider these issues.*

So HAS anyone proven if the countable model of ZF consistent or not?  
And what are you saying? That logicians, like mathematicians already  
have my interests at heart, and that I should shut up and leave this  
to experts? That

- > /*The Berry Paradox, is not well–defined in my book, since "describable*
- > *in less than forty english words" is not well formed. There is no*
- > *such number, and there is no proof that such a number exists.*
- >
- > *Exactly. Just as there's no countable set containing all numbers*
- > *definable in any way whatsoever.*

I agree that there is no SET of all definable numbers. There could be  
a CLASS of all definable numbers, but then the statement that there  
must be a smallest member of the SET of all definable numbers comes  
crashing down.

I'm not an expert on forcing, and I don't know the name of the  
countable model of ZF. What IS the class of sets that have a  
representative in the countable model that is interpreted as the set?  
Does it HAVE a name?

J.E.