Re: Another set with cardinality |Z|

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erayo@bilkent.edu.tr (Eray Ozkural exa) wrote in message
news:<fa69ae35.0409221823.682ae186@posting.google.com>...
> Let's have an algorithm that starts with
> 0.1 in binary, and constructs a tree in breadth−first fashion
>
> 0.1
> 0.01 0.11
> 0.001 0.011....
>
> You get the idea... It's obvious that this tree has the same
> cardinality as Z, since this is a nonhalting algorithm (or since I can
> give an integer to every node, etc.) Now, I want to prove that such a
> subdivision procedure cannot generate all x in (0,1) in an intuitive
> way. Is the easiest method proof by contradiction?
>
> Regards,

All of the numbers you have generated are of finite length, can you find some numbers who's binary representations never terminate? Finding counter−examples seems to me to be a form of proof by contradiction although I find it convienent to distinquish between the two since counter−examples are sometimes easier to find then a deduced logical flaw, although essentially I think that's what you're doing.

If you're looking for a mapping from your set to the subset of naturals you could try taking the set of primes and mapping them like so, 0.1 = 2^1, 0.01 = 3^1, 0.11 = 2^1 * 3^1 and so on, giving a unique prime to each binary place and making each of the numbers your algorithm generates match up with a unique number. of the form p_1^{0 or 1} * p_2^{0 or 1} * ... * p_n^{0 or 1} where p_k is the k−th prime number and n is the breadth of the number and you select 0 or 1 based on which binary digit is in the k−th place of the number. You could then refer to Cantor's proof of the uncountability of the interval (0,1). Mapping in a similar fashion to powers of primes is an easy way to show that the cartisan product of two countable sets is also countable, by mapping it to a subset of N and extends easily to the case of n cartisean products. Hope this helps. :)