

Re: Simple matrix proof

Source: <http://sci.tech-archive.net/Archive/sci.math/2004-09/6643.html>

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Date: 09/27/04

Date: Mon, 27 Sep 2004 20:22:02 +0000 (UTC)

Thank you all for your input. I figured out how to do it somewhat properly through $MX=0 \Leftrightarrow X=0$ by considering that you can always choose k so that there is at least one $A^{(k-1)}$ which is not zero. However, the series approach was a complete and nice surprise after a day of work with the problem :)

joccis

On 26 Sep 2004, Alain Verghote wrote:

>On 26 Sep 2004, joccis wrote:

>>Let A be a $(n \times n)$ matrix, such that $A^k=0$ for some $k>1$.

>>Show that $I-A$ is non-singular.

>>

>>Proof

>>Let $M=I-A$, then

>> $A^{(k-1)}M = A^{(k-1)}I - A^{(k-1)}A^k$

>> $A^{(k-1)}M = A^{(k-1)} - A^k$

>> $A^{(k-1)}M = A^{(k-1)}$

>> $\Leftrightarrow M=I$, hence M is non-singular.

>>

>>Am I missing something trivial here? If $M=I$, then $A=0$, and $A^k=0$ on every k and the statement turns out quite uninteresting.

>>

>>regards

>>joccis

>

>Dear joccis,

>

>We choose $k = n$ and $A^n=0$.

> Proof $I/(I-A)$ exists .Because $A^n=0; I/(I-A)=I+A+A^2+..A^{(n-2)}+A^{(n-1)}$

> Look you don't need bigger power than $n-1$.

> If $(I-A)^{-1}$ is defined then $I-A$ is non-singular.

> Hope It helps.

>

>Alain.