Re: Attempt epsilon−delta

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From: Chris Wagner (*clwagner_at_vulcan.wagner.nul*)
Date: 10/01/04

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Date: Fri, 1 Oct 2004 12:25:14 +0000 (UTC)
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In article <cjevb0$1q9h$1@agate.berkeley.edu>,
magidin@math.berkeley.edu (Arturo Magidin) writes:
> *In article <cjeohn$3cvr$1@murrow.it.wsu.edu>,*
> *Chris Wagner <clwagner@vulcan.wagner.nul> wrote:*
>>*Greetings,*
>>
>>*Here is my attempt at an elementary epsilon−delta proof.*
>>
>>*Prove lim_{x−>a} f(x) = f(a), for continuous f(x) in Reals.*
>
> *What is your definition of continuity and of limits? I ask, because*
> *sometimes "continuous" is defined to mean "the function is defined at*
> *the point and the value of the limit exists and agrees with the value*
> *of the function", which would clearly make this a tautological*
> *statement.*

It did seem a bit tautological to me. My purpose was to simplify so I
could understand epsilon−delta well enough to construct proofs. It is
my understanding that the Cauchy−Weierstrass epsilon−delta method is
considered definitive, that is, if I can correctly do e−d I can Prove
claims about limits.

>
> *I assume that you are using the following definitions for limit and*
> *for "continuity at x=a":*
>
> *DEF. Let f(x) be a function. The limit as x goes to a of f(x) is equal*
> *to L, lim_{x−>a} f(x) = L if and only if*
>
> *for every e>0 there exists d>0 such that*
>
> *if 0< | x−a | < d, then |f(x)−L|< e.*
>
>
> *DEF. Let f(x) be a function. Then f(x) is continuous at a if and only*
> *if f is defined at a, and*
>
> *for every e>0 there exists d>0 such that*

>
> *if |x−a| < d, then |f(x)−f(a)|<e.*
>

These are indeed the e−d definitions I hope to apply. I read the definitions in Rudin's "Principles of Mathematical Analysis" (third ed.), begining of chapt 4.

Perhaps I can state my problem directly from the two definitions quoted above:

Given continuous function f(x) and e>0 what is d? I need to understand how to construct the d (my response to your e challenge), since that seems to be the essence of doing such proofs.

>>*Let*
>>*epsilon>0 be given. Using the Mean Value Theorem (MVT)*
>
> *I'm the one who usually uses cannonballs to swat flies... but isn't*
> *this a bit beyond that?*

Urk−gulp.

[snip]
>
> *Look at the definitions above. Can you not see why if the second one*
> *holds, then the first one will hold with L=f(a)?*

OK, if the second definition holds then we know there is a delta. But what is it? (d as function of f and e)

>
[snip]
>>*Is my attempt acceptable or salvageable?*
>
> *I would consider it completely unacceptable.*
>

Thank−You for your reply to my post.

[snip]
>>*Does not continuity mean lim_{x−>a}f(x)=f(a),*
>
> *Depends on the definitions.*
>
>> *without requiring*
>>*inverse f or differentiability of f assumption?*
>
> *Correct. Look at the definitions you have.*
>

OK, Good.

Thanks,
Chris Wagner