Re: Complex differentiable and multiplication

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On Sat, 2 Oct 2004 11:37:56 +0000 (UTC), anonymous@mathforum.org
(Sergei) wrote:

>
> Could someone explain to me the meaning of this statement:
>
> (From Mathworld)
>
> Re: Complex−differentiable functions:
>
> "...Then f is complex−differentiable if its' Jacobian is
> of the form:
>
>
> [a b] (*)
> [−b a]
>
> ...."
>
> This follows from C−R, but , below is where I am not clear:
>
> That is, the derivative is given by the multiplication of
> a complex number".

The derivative they're talking about here is a real−linear map
from R^2 to R^2 given by that matrix of partial derivatives.
Say T : R^2 −> R^2 is real−linear. Then it's "given by multiplication
by a complex number" if and only if it's equivalent to a complex−
linear map from C to C (equivalent using the standard identification
of C with R^2). And T is complex−linear if and only if it commutes
with multiplication by i: T(iz) = iT(z).

Now say z = x + iy, T is defined by the matrix

[a b]
[c d],

note that i(x,y) = (−y,x), and verify that T commutes with
multiplcation by i if and only if c = −b and d = a.

> *I understand that there is an isomorphism from C to*
> *GL(4,R) given by a+bi−>[a b] .*
> *[−b a]*
>
> *And I know that complex multiplication implies both*
> *scaling and rotating, but how does the above representation*
> *of the Jacobian (*) relate analytic functions with*
> *multiplication?*
> *Thanks for any help.*
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David C. Ullrich