

Re: Nowhere continuous function

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In article <93b1dfbf.0410021553.7ef07e52@posting.google.com>, artur@opendf.com.br (Artur) wrote:

> *Hello*

>

> *I have a question: Can there exist a function $f: I \rightarrow R$, I an open interval of R , which has a limit at every point of I but is nowhere continuous?*

No, in fact the set of discontinuities is at most countable. Here's a thread where this was discussed:

<http://snipurl.com/9i2m>

One way to proceed is to let $g(x)$ be the limit of f at x ; so $g : I \rightarrow R$. Let $D_n = \{x \text{ in } I : |f(x) - g(x)| > 1/n\}$. The set D of discontinuities of f is the union of the D_n 's. If D were not countable, then some D_n would be uncountable, hence would have an accumulation point p somewhere in I . Now check that $\lim_{x \rightarrow p} f(x)$ fails to exist, contradiction.