

Re: prime~.

Source: <http://sci.tech-archive.net/Archive/sci.math/2004-10/1187.html>

From: L.B. (*metafert_at_mail.ru*)

Date: 10/04/04

Date: 4 Oct 2004 06:53:25 -0700

"mina_world" <mina_world@hanmail.net> wrote in message news:<cjqd4r\$c9u\$1@news.hananet.net>...

> "Michael Lockhart" <ml1000@bellsouth.net> wrote in message
> news:X828d.227877\$%n4.3977@bignews6.bellsouth.net...

>> "mina_world" <mina_world@hanmail.net> wrote in message
>> news:cjq9ds\$a7o\$1@news.hananet.net...

>>> hello.....doctor~

>>>

>>> *p* is a prime.

>>> *n* is positive integer.

>>>

>>> show that

>>> $p \mid (a^n) - 1$ for some a in Z

>>> $\Rightarrow (p^2) \mid (a^n) - 1$

>>

>> That's not true.

>>

> oh.....i'm sorry. my mistake.

>

> *p* is a prime.

>

> show that

> $p \mid (a^p) - 1$ for some a in Z

> $\Rightarrow (p^2) \mid (a^p) - 1$

>

> it's exact.

>

> Pardon me. sorry.

Let $a^p \equiv 1 \pmod p$. From other side $a^{(p-1)} \equiv 1 \pmod p$, Fermat's theorem.
 So, we get $a \equiv 1 \pmod p$. Let $a = 1 + p*b$. Then

$$a^p - 1 = (1 + p*b)^p - 1 = 1 + \sum_{i=1}^p \binom{p}{i} (p*b)^i - 1$$

$$= \sum_{i=1}^p \binom{p}{i} (p*b)^i. \text{ As is well know } - \binom{p}{i} \equiv 0 \pmod p$$

and obviously $(p*b)^i \equiv 0 \pmod p$, since

$$a^p - 1 = \sum_{i=1}^p \binom{p}{i} (p*b)^i \equiv 0 \pmod{p^2}$$