

Re: um.....this is...!

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"Ignacio Larrosa Ca?stro" <ilarrosaQUITARMAYUSCULAS@mundo-r.com> wrote in message news:2sfst4F1kkvtbU1@uni-berlin.de...

> *En el mensaje:cjudam\$gor\$1@news.hananet.net,*
> *mina_world <mina_world@hanmail.net> escribi?*
> > *hello.....doctor~*

> >

> > $a_1 = 1$

> >

> > $2\{a_{(n+1)}\} = 3\cdot a_n + \text{sqrt}\{(5(a_n)^2) + 4\} \ (n \geq 1)$

> >

> > *show that there does not exist n such that $1989 \mid a_{(2n)}$.*

> > *(n is positive integer)*

> >

> > -----

> >

> > *um.....it's so difficult to me.*

> >

> > *i need your advice.*

> >

> > *thank you very much for your advice.*

>

> *Hint: Prove that $a(n) = \text{Fibonacci}(2n)$*

>

>

um.....

$a_1 = 0$

$a_2 = 1$

$a_3 = 3$

$a_4 = 8$

$a_5 = 21$

$a_6 = 55$

$a_{(n+1)} - 3\cdot a_n + a_{(n-1)} = 0$ by rule.

and

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$$\begin{aligned} a_n &= (1/\sqrt{5}) * \{ ((3+\sqrt{5})/2)^{n-1} - ((3-\sqrt{5})/2)^{n-1} \} \\ &= (1/\sqrt{5}) * (2/(3+\sqrt{5})) * \{ ((3+\sqrt{5})/2)^n - ((3-\sqrt{5})/2)^n \} \end{aligned}$$

by characteristic equation.

and

$$\begin{aligned} a_{2n} &= (1/\sqrt{5}) * (2/(3+\sqrt{5})) * \{ ((3+\sqrt{5})/2)^{2n} - ((3-\sqrt{5})/2)^{2n} \} \\ &= (1/\sqrt{5}) * (2/(3+\sqrt{5})) * \{ ((14+6.\sqrt{5})/4)^n - ((14-6.\sqrt{5})/4)^n \} \end{aligned}$$

and

let $a_{2n} = b_n$
 $b_{n+2} = 7.b_{n+1} - b_n$ by reverse pursuit to characteristic equation.

and

$$\begin{aligned} b_1 &= a_2 = 1 \pmod{9} \\ b_2 &= a_4 = 8 \pmod{9} \end{aligned}$$

and

suppose that
 $b_{2n-1} = 1 \pmod{9}$
 $b_{2n} = 8 \pmod{9}$

$$\begin{aligned} b_{2n+1} &= 7.b_{2n} - b_{2n-1} = 7.8 - 1 = 55 = 1 \pmod{9} \\ b_{2n+2} &= 7.b_{2n+1} - b_{2n} = 7.1 - 8 = -1 = 8 \pmod{9} \end{aligned}$$

by mathematical induction,
 $b_{2n} = 8 \pmod{9}$
 $b_{2n+1} = 1 \pmod{9}$

but, $1989 = 0 \pmod{9}$

thus, $1989 \mid b_n = a_{2n}$ is impossible.

um.....is it insufficient ??

thank you very much.