

Re: how to get the angle from the cosine, etc.

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"Sean Hunt" <seanstewarhunt@hotmail.com> wrote in sci.math:
> *I'm actually looking for a way to get the angle (in radians), given*
> *that you already have the sine and cosine of the angle. I'm trying to*
> *derive an equation to go back and forth rather than a look-up process,*

What do you mean by "an equation"? I'm guessing that you mean some sort of polynomial, or at worst something involving only powers, multiplication/division, and addition/subtraction. No such equation can exist, because $\cos(\)$ is a transcendental function and so is its inverse.

The only finite equation that is an answer to your question is
 $\text{angle} = \arccos(x)$

where x is the cosine. (\arccos is sometimes written as \cos with a superscript -- not exponent -- of -1 .)

What we are forced to do is to define the inverse cosine as a new function. But since cosine is a many-to-one function, inverse cosine would be a one-to-many function. Therefore we pick an interval and say that $\arccos(x)$ is always between 0 and π . This has the good effect that $\arccos(\)$ is now a function, but it does mean that we can't write $\arccos(\cos(\theta)) = \theta$.

There are some issues with that. For instance, if the cosine is $1/2$, you might be tempted to say that the angle is $\pi/3$. And indeed $\arccos(1/2) = \pi/3$. But many other angles also have a cosine equal to $1/2$. The original angle could also be $5\pi/3$, or $2\pi + \pi/3$, etc. If you also know the sine, then you have a unique solution within the interval $[0, 2\pi)$, but overall there are still an infinite number of solutions.

> *On the surface it looks like a fairly easy correlation, but I*
> *haven't been able to calculate the angle given sine and cosine. Now*
> *I'm beginning to think that this might be a calculus problem rather*
> *than a trigonometry problem.*

Well, if you're willing to accept an infinite series, the calculus

sci.math: Re: how to get the angle from the cosine, etc.

can help. But no closed-form solution exists.

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Fortunately, I live in the United States of America, where we are gradually coming to understand that nothing we do is ever our fault, especially if it is really stupid. --Dave Barry