Re: Zenkin's paper on Cantor

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daryl@atc−nycorp.com (Daryl McCullough) wrote in message news:<ck26jh030rb@drn.newsguy.com>...
> Eray Ozkural exa says...
> >
> >Ralph Hartley <hartley@aic.nrl.navy.mil> wrote
>
> >> If you give me a list of real numbers, presented in that way, I can give
> >> you a number not on your list.
> >
> >A timely observation which takes us to the heart of the matter. I will
> >argue that I cannot "give you a list".
>
*> You can give me a *procedure* which, given a number n*
> returns the nth real number (between 0 and 1, for simplicity),
> which in turn is a procedure which, given a number m, returns
> the mth bit of the binary expansion of that real.

Yes, of course, Daryl, that is quite obvious. I can for instance use a regular subdivision procedure which, given a number n, returns the number "0.binary(n)".

I can also guarantee that the nth number has at least n digits after the (binary) point (I think this is easily done). But the above method doesn't hold then, you have to use something like "0.<n 0's>1".

Now, giving the list in this fashion would be potential infinity, since it is compressed in a finite nonhalting program. My post is in error, the second proof also can be said to be valid from a constructivist perspective, if the reals chosen are deliberately taken to be computable in exactly this way.

I think this way, the essence of the proof is actually demonstrated better; note the similarity of my representation above to the fact that the measure of the countable reals in (0,1) is 0. (but of course different representations are possible)

I am pleased with this interpretation of the proof. The later steps of the proof can be merged in a nonhalting procedure which uses the generator above, the reasoning part is still infinitary, but I think it is inductively true (I can try to explain this part if it doesn't make sense to you). Now, everything is explained bottom−up, when we are dealing only with a *computable* list of computable reals. (The actual infinity remains in the continuum, the proof has nothing to do with it.)

But I do not think that Cantor's original proof is in fact constructive in the sense of *constructivism*. (I'm aware that a realist might call it constructive!)

Regards,

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Eray Ozkural
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