Re: 3^k + 2^k revisited

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Date: 10/07/04

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Am 07.10.04 09:09 schrieb William Elliot:

> From: Gottfried Helms <helms@uni−kassel.de>
> Newsgroups: sci.math
> Subject: Re: 3^k + 2^k revisited
>
> schrieb Ray Steiner:
> >>A while ago the problem of whether 3^k + 2^k could be a power for
> >>odd k was posed to this group. I would like to propose a special
> >>case: If k is odd, can 3^k + 2^k ever be a square?
>
> > let
> > 3^k + 2^k = s^2 (1)
>
> > then s^2 == 1 (mod 3)
> > 2^k == 1 (mod 3) if k is even.
> > 2^k == 2 (mod 3) if k is odd.
>
> > So for odd k (1) has no solution in integer k,s>0, k odd
> > (but what about even k?)
>
> 3^k + 2^k is never a square, thus never an even power.
>
> Case: k = 2j + 1
> 3^k + 2^k = 9^j 3 + 4^j 2 = 0 + 1^j 2 = 2 (mod 3)
> 0^2 = 0, 1^2 = 1, 2^2 = 1 (mod 3)
>
> Case: k = 2j
> 3^k + 2^k = 9^j + 4^j = (−1)^j + (−1)^j = +−2 (mod 5)
> 0^2 = 0, 1^2 = 1, 2^2 = −1, 3^2 = −1, 4^2 = 1 (mod 5)
>
> −−−−
>
>

Well, that's concise...

I stuck with the following

since
    3^k −1 = 2^a *u // u odd
and
  a k
−−−−−−−−−−−−−−−−−−−−−−−−−−−−−−−−−−−−−
  3 2+4*i = 2,6,10,14,18,....
  4 4+8*i = 4,12,20,...
  5 8+16*i = 8,24,...
  ...
−−−−−−−−−−−−−−−−−−−−−−−−−−−−−−−−−−−−−
  3^k − 1 + 2^k = s^2 − 1

  2^a (u + 2^b) = (s+1)(s−1) // b = k−a

I first thought, "a" must be even, and so k must be divisible by 4 (plus some more restrictions) and the right han