

Re: Derivative of exponential function

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In article <200410071712.i97HCAL62738@mickey.empros.com>, mstemper@siemens-emis.com (Michael Stemper) wrote:

> I'm in the process of home-schooling my son in calculus. We're about to
> finish up on limits, so that we can start differentiation. I've relearned
> enough that I've been able to derive, from the definition of derivative,
> the derivatives of c , x , cx , $f(x)+g(x)$, $f(x)*g(x)$, $f(g(x))$, $\sin(x)$,
> $\cos(x)$. From those, I can get the derivatives of $f(x)-g(x)$, $f(x)/g(x)$,
> $(f(x))^n$, $(f(x))^{-n}$, $\tan(x)$. A pretty solid beginning. But there's
> one big hole: I can't prove that the derivative of e^x is e^x .
>
> I can see that if $\lim_{h \rightarrow 0} ((e^h - 1)/h) = 1$, then I'm set. Numerically,
> I can crank out values for $((e^h - 1)/h)$ with h getting very small, and
> see that they get really close to 1. But, that's not proof.
>
> I was going to base a proof on the Taylor series for e^x , but that
> depends upon already knowing the n th derivatives of e^x , so that was
> out. Then I was going to try using my knowledge of the behavior of
> e^x , but upon examination, that was all based on knowing things about
> its derivative.
>
> Is there some simple (or subtle) trick that I'm overlooking? Is the
> proof of this limit actually incredibly hard?
>
> I can't look in the book, because it's with him (he lives with his
> mother). I've even tried typing " $((e^h - 1)/h)$ " into Google, but that
> just turned up a bunch of PDF files. Any help, or am I going to need
> to do some serious hand-waving?

How did you define e^x in the first place?

The two commonest definitions, at least as far as I recollect, are
(1) by the power series and (2) as the inverse function to the natural
logarithm function (in turn defined as $\log(x) = \int_{t=1..x, 1/t}$,
for $x > 0$).

Case (1) you say you know how to do.

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In case 2, apply the chain rule to $x = \log(e^x)$

If you have some other definition, if you post your definition, maybe someone will be able to help.