Re: Derivative of exponential function

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In article <200410071712.i97HCAL62738@mickey.empros.com>, Michael Stemper <mstemper@siemens−emis.com> wrote:

> I'm in the process of home−schooling my son in calculus. We're about to
> finish up on limits, so that we can start differentiation. I've relearned
> enough that I've been able to derive, from the definition of derivative,
*> the derivatives of c, x, cx, f(x)+g(x), f(x)*g(x), f(g(x)), sin(x),*
> cos(x). From those, I can get the derivatives of f(x)−g(x), f(x)/g(x),
> (f(x))^n, (f(x))^(−n), tan(x). A pretty solid beginning. But there's
> one big hole: I can't prove that the derivative of e^x is e^x.

What is your definition of e^x ? There are many possibilities,
and proofs of facts about e^x depend on it. If you cannot
define e^x (for real x, including irrational x), then of course
you cannot prove anything about that function.

>
> I can see that if Lim(h−>0) ((e^h−1)/h) == 1, then I'm set. Numerically,
> I can crank out values for ((e^h−1)/h) with h getting very small, and
> see that they get really close to 1. But, that's not proof.
>
> I was going to base a proof on the Taylor series for e^x, but that
> depends upon already knowing the nth derivatives of e^x, so that was
> out. Then I was going to try using my knowledge of the behavior of
> e^x, but upon examination, that was all based on knowing things about
> its derivative.
>
> Is there some simple (or subtle) trick that I'm overlooking? Is the
> proof of this limit actually incredibly hard?
>
> I can't look in the book, because it's with him (he lives with his
> mother). I've even tried typing "((e^h−1)/h)" into Google, but that
> just turned up a bunch of PDF files. Any help, or am I going to need
> to do some serious hand−waving?

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