Re: Distance Between 2 Randomly−chosen Points on a Sphere

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From: David Kastrup (*dak_at_gnu.org*)
Date: 10/09/04

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Date: Sat, 09 Oct 2004 21:56:20 +0200
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norabaron@hotmail.com (Nora Baron) writes:

> David C. Ullrich <ullrich@math.okstate.edu> wrote in message
news:<asjfm0dokt0bgaq7gre7hp8vfm2kpkdq5s@4ax.com>...
>> On 8 Oct 2004 12:28:03 −0700, norabaron@hotmail.com (Nora Baron)
>> wrote:
>>
>> >"Eamon Warnock" <ewarnock@gz.cngb.com> wrote in message
news:<00c9e4d0a5dcef258d26cda4af481189.61944@mygate.mailgate.org>...
>> >> What is the most likely straight−line distance between two randomly
>> >> selected points on a sphere?
>> >
>> > Two randomly chosen points lie on one unique great circle
>> >(with probability 1).
>>
>> Yes.
>>
>> >You can without loss of generality
>> >assume that circle is in the x−y plane and that one of the
>> >points has coordinates (1, 0, 0).
>>
>> Yes.
>>
>> > The other point has a
>> >uniform probability of being anywhere on the circle.
>>
>> No. Say one point is (1,0,0), and say E is the equator.
>> Say p is the actual other point, and p' is "the" point
>> on E which is at the same distance from (1,0,0). Say
>> d > 0 is small.
>>
>> The probability that d' lies on an arc on the equator
>> of length d which includes the point (1,0,0) is
>> proportional to the area of a spherical cap of
>> radius d, or roughly d^2. Otoh if you consider an
>> arc on the equator of length d which passes through

>> the point (0,1,0) the probability that p' lies on
>> that arc is the probability that p lies in a
>> sort of strip of width d, which is roughly a
>> constant times d.
>>
>> > The
>> >distance, however, is not uniformly distributed, and clearly
>> >has highest density when the second point is as far as possible
>> >from the first.

Uh, what?

>> >That is, the "most likely" distance is the diameter. The
>> >least−likely distance is zero. The expectation of the distance is
>> >(2/3) * diameter.
>> >
>
> No − my argument is correct − first, you know that the two
> points are on a unique great circle. Then assume that one
> of the points is *labelled* as (1, 0, 0). *Conditional on
> both of these*, the second point has a uniform distribution
> on that great circle. The rest follows.

Uh, what? What, what, what?

This appears all like complete bull to me, sorry. If I choose 2 points randomly and independently, then I can without loss of generality put one point on the north pole. Integrating the distance theta over the sphere in sphere coordinates gives for the expected distance

1/(4pi) int 0..pi dtheta int 0..2pi dphi sin(theta) theta = pi/2

which is half the great circle. Hardly surprising, considering that for every point at a distance of x from the pole, there is a point mirrored at the equator at distance pi−x.

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−−
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