Re: before Cantor

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"David Bernier" <ezcos@yahoo.com> wrote in message
news:50dce0c5.0410112116.7afacf7f@posting.google.com...
> *Today, speaking of mathematical objects that exist*
> *but cannot possibly be defined is accepted by a large*
> *majority of mathematicians; also existence proofs*
> *that I know of for Hamel basises and non−measurable subsets*
> *of the reals don't "really" define them.*
>
> *Before Cantor, did anyone speak of objects (not variables...)*
> *that were not defined in the usual sense?*
>
> *(I might want to go over the proof of the Riemann mapping*
> *theorem in complex analysis.)*
>
>
> *David Bernier*

Hamel bases and non−measurable functions exist in the universe of ZFC−−Zermelo−Frankael set theory with Axiom of Choice (AC)−−but not in ZF (without AC). The "mainstream" set−theoretical position is that questions about mathematical "existence" of an object are to be treated as just questions about which axioms of set theory are being used to show that the object is a "set".

Be careful with how the word "exist" is being used in mathematics as questions about the "existence" of mathematical objects are too often loaded and emotionally biased−−usually on an unconscious level−− with extra−mathematical imports. Instead of asking "does it really exist", ask "where does it exist?" Where do all mathematical objects exist? Surely, they "exist" in a rational mind in the sense of being capable of being conceptualized there.

One of the most ingenious (some might say, expedient) axioms of ZF set theory is the Axiom of Extensionality (AE). It "cuts to the chaste" by side−stepping metaphysical and ontological issues of "existence" which are, perhaps, best left for philosophers and theologians when determining "set equality". Alternatives to AE (such as "intensionality") have been explored, but most, if not all, would agree that mathematics has no need for an alternative to AE.

One does not have to look so far to objects as non−measurable functions or a Hamel basis to see the point that mathematical objects "exist" ONLY in the sense of being a "set" in the underlying set of axioms being used. Where does "five−ness" or "eight−ness" exist? Fiveness is a distillation of "five oranges", "five apples", and so on. One may be able to convince oneself that the five oranges in the refrigerator exist in the physical universe, but does it follow that "fiveness" exist in the physical universe as well? I think not.

Where does the number "0" exist? Where does the "imaginary" unit "i" (where $i^2 = -1$) exist? Does "i" have less "real existence" (whatever that means) than "fiveness" or "eightness"? Are we not justified to believe in "i" as a solution to the equation ($x^2 = -1$) just as much as we must believe in "5" and "8" as solutions to the congruence relation $x^2 = -1$ under modulo 13 integer arithmetic?

> *Before Cantor, did anyone speak of objects (not variables...)*
> *that were not defined in the usual sense?*

Remember Leibniz tried to justify the "existence" of his use of "infinitesimals" in calculus by appealing to his set of philosophical principles (Leibniz was a philosopher among many things)? He was severely criticized by Berkeley who called them "ghosts of departed quantities". Of course, nowadays, we have no need for them in standard analysis−−thanks to Cauchy and Weierstrass. [P.S. In the 1960s, A. Robinson resurrected "infinitesimals" in non−standard analysis on rigorous mathematical grounds. Therefore one is now able to believe in the "existence" of infinitesimals without losing mathematical rigor. No philosophy is required, though.]

Shedar