Re: Exponential RV and Conditional Expectation Problem

Source: http://sci.tech−archive.net/Archive/sci.math/2004−10/4184.html

---

From: Robert Israel (*israel_at_math.ubc.ca*)
Date: 10/14/04

Date: 14 Oct 2004 17:20:50 GMT

In article <216501ce.0410140209.1edcfdb3@posting.google.com>,
bg <Stridar@gmail.com> wrote:

>I am working alone through a book (Sheldon Ross's _Probability Models
>for Computer Science_). While I have enjoyed the exposition and have
>been able to do most problems, I do not understand the math behind one
>question. Would anyone be kind enough to show me how to calculate the
>conditional expectation in the following problem?

>Problem 1.23

>Let X,Y be ind. exponential r.v.'s with rates \lambda and \mu, and let
>c >= 0.

>a) Find E(min(X,Y) | X > c)
>b) Find E(min(X,Y) | X > Y + c)

>For part (a), I have tried breaking the integral into three parts to
>remove the min function instead of finding a joint probability
>density. Is this a correct method?

Yes. You might also break it into two pieces Y <= c and Y > c, and
for the second piece use the facts that
1) the conditional density for X − c given X > c is the same as the
density for X [and similarly for Y]
2) the minimum of independent exponential r.v.'s is an exponential r.v.

>For part (b), I am lost. I thought conditioning on Y would work since
>it seems min(X,Y) should simply be Y instead of integrating over the
>exponential r.v. with parameter \mu+\lambda. However, Ross gives a
>later problem to show
> E(min(X,Y) | X > Y + c) = E(min(X,Y)| X>Y) = \frac{1, \lamda +
>\mu}
>The above technique does not evaluate to this fraction.

Yes, min(X,Y) = Y when X > Y + c. By "conditioning on Y" I hope you mean

E[Y | X > Y + c] =
int_0^infty y P{X > y + c} f_Y(y) dy / int_0^infty P(X > y + c) f_Y(y) dy

Robert Israel israel@math.ubc.ca
Department of Mathematics http://www.math.ubc.ca/~israel
University of British Columbia Vancouver, BC, Canada