Re: Exponential RV and Conditional Expectation Problem

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Stridar@gmail.com (bg) wrote in message news:<216501ce.0410140209.1edcfdb3@posting.google.com>...
> *Dear all,*
>
> *I am working alone through a book (Sheldon Ross's _Probability Models*
> *for Computer Science_). While I have enjoyed the exposition and have*
> *been able to do most problems, I do not understand the math behind one*
> *question. Would anyone be kind enough to show me how to calculate the*
> *conditional expectation in the following problem?*
>
> *Problem 1.23*
>
> *Let X,Y be ind. exponential r.v.'s with rates \lambda and \mu, and let*
> *c >= 0.*
>
> *a) Find E(min(X,Y) | X > c)*
> *b) Find E(min(X,Y) | X > Y + c)*
>
>
> *Thank you,*
> *BG*
>
> *P.S.*
> *For part (a), I have tried breaking the integral into three parts to*
> *remove the min function instead of finding a joint probability*
> *density. Is this a correct method?*

If done correctly. What are your three parts?

I'd do it this way:
Think in terms of a new rv, Z = min(X,Y). You can easily
reason about the cumulative distribution of Z.

Prob(Z<z|X>c) = 1 − P(Z>=z | X>c). I find this latter
probability easier to reason about. Z>=z iff both X>=z and
Y>=z. Thus

P(Z>=z | X>c) = P(Y>=z & X>=z|X>c)
    = P(Y>=z|X>c) P(X>=z|X>c)

You know how to evaluate those expressions.

This is in general a useful trick for dealing with the
minimum.

> *For part (b), I am lost. I thought conditioning on Y would work since*
> *it seems min(X,Y) should simply be Y*

That seems correct.

> *instead of integrating over the*
> *exponential r.v. with parameter \mu+\lambda. However, Ross gives a*
> *later problem to show*
> *E(min(X,Y) | X > Y + c) = E(min(X,Y)| X>Y)*

I have to think about this one a little.

> *= \frac{1, \lamda +*
> *\mu}*

While it may work out to this, I don't think it's because
there is an exponential rv with parameter lambda +mu.

− Randy